

# THE MINISTRY OF PRODUCTION IN THE COLLECTIVIST STATE\*

ENRICO BARONE

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## I. THE SCOPE OF THIS ARTICLE

1. In the consideration of production in a collectivist State there are two questions entirely distinct from each other. The first is: Will it be beneficial for some of the capital<sup>1</sup> to become collective property and for production to be socialized? The second is this: How, in a collectivist régime, ought production to be directed? One can discuss the second question quite independently of the answer one gives to the first. My particular purpose here is to make a study of the second question, setting the problem in as precise a form as is possible.

Hence I do not write for or against Collectivism. I assume it to be established in a certain social group and I propose to establish certain general lines of the solution which the Ministry of Production ought to give to the vast problem with which it is faced.

Many believe that they have confuted Collectivism when they have shown that some propositions, of Marx or of others, contain errors and contradictions. But the mere confutation of these propositions has not, in fact, any value, because without falling into such errors and contradictions one can very well imagine an economic system which would realize the spirit of the Marxist system. Logical absurdities can be eliminated. But it is necessary to have a clear idea of what the nature of the system could be after eliminating such absurdities. The elucidation of this system is the object of the following pages.

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\* "Il ministro della produzione nello stato collettivista", *Giornale degli Economisti*, 37 (September, October 1908): 267-293, 391-414. Abridged English translation in F.A. Hayek (ed.), *Collectivist Economic Planning*, London: Routledge & Kegan Paul, Ltd., 1935: 245-290. The name of the translator is not indicated.

<sup>1</sup> [The term capital is here and throughout this article used in the comprehensive sense introduced by Professor Irving Fisher. It includes land as well as the produced means of production. - *Ed.*]

2. In this article I use mathematics for the simple reason that I do not know another method which, with similar precision and brevity, allows me to put certain questions in unequivocal terms and to give a precise exposition of certain propositions...<sup>2</sup>

3. Since many, who speak of arguments which they do not understand, show that they believe that the Mathematical School and the Austrian School are identical and that the former must necessarily make use of some of the fundamental concepts of the latter, I propose to prove also, that to define the economic equilibrium – be it in a régime of free competition, in one of monopoly, or in the Collectivist State – there is no need to have recourse to the concepts of *utility*, of the *final degree of utility*, and the like; and neither is it necessary to have recourse to Pareto's concept of the *Indifference Curve*, although it represents a notable step in freeing the Mathematical School from all that seems metaphysical. The old and simple ideas of demand, supply and cost of production, suffice, not only to construct into a system of equations the most important interrelations of economic quantities, but also to treat the various dynamic questions which relate to the greater or smaller welfare of individuals and of the community.

4. In this article – in which I have used freely the works of my predecessors, and especially that of Vilfredo Pareto, to which I have added my original contribution – I propose to determine in what manner the Ministry concerned with production ought to direct it in order to achieve the maximum advantage from its operations. Some of the arguments I use and some of the conclusions at which I arrive have already been made available to us, as the special contribution of the indefatigable and prolific work of that solitary thinker of Céligny. Others are my own. This I say not in order to draw attention to the original element in my work. Rather, it is my purpose to make sure that readers little familiar with the new theories should not attribute to me that which belongs to Walras and Pareto.

## II. THE INDIVIDUALIST RÉGIME

5. *The Data and the Unknown Quantities.* – This régime is essentially one in which free competition, monopolies and cartels are all present.

Let us state the conditions of equilibrium, dealing first with free competition, afterwards introducing monopolies and cartels.

The data are: the quantity of capital (including free capital) possessed by each individual; the *relations*, in a given state of technique, between the quan-

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<sup>2</sup> [In the passages which were left out Barone referred to a further instalment of this article in which he intended to present the problem in the form of a discourse which the Minister of Production of the Socialist State delivers to his colleagues. This part was unfortunately never published. - Ed.]

tity produced and the factors of production; and the *tastes* of the various individuals. On these last we will make no pre-supposition, no preliminary inquiry, limiting ourselves simply to assuming the fact that at every given series of prices of products and productive services, every single individual portions out the income from his services between consumption and saving in a certain manner (into the motives of which we will not inquire) by which, at a given series of prices, the individual makes certain demands and certain offers. These quantities demanded and offered vary when the series of prices vary.

Thus we disengage ourselves from every metaphysical or subtle conception of utility and of the functions of indifference, and rely solely on the authenticity of a fact<sup>3</sup>.

6. Let us represent among the data the quantities of the different kinds of capital possessed by single individuals. Let the different kinds of capital be  $S, T \dots$  to  $n$  terms. The total quantities of these existing in the group will be  $Q_s, Q_t \dots$ . Among these  $n$  kinds of capital there is also working capital, and also the kinds  $H, K \dots$  (to  $n$  terms) of new capital in process of construction.

Let the technical coefficients be  $a_s, a_t, \dots, b_s, b_t \dots$ , indicating, respectively, the quantity of services  $S, T \dots$  necessary for the manufacture of every unit of  $A, B \dots$  which are the various kinds of products,  $m$  in number.

For the present we will not count the technical coefficients among the unknown; let us suppose them given, temporarily. We shall see afterwards that they are determined by the condition of minimum cost of production.

The unknowns are set out in the following table:

	Quantity	Number of Unknowns
<i>Products:</i>		
Quantity demanded and produced	$R_a, R_b, \dots$	$m$
with cost of production	$\pi_a, \pi_b, \dots$	$m$
and prices	$1, p_b$	$m - 1$
<i>Existing Capital:</i>		
Quantity of their services directly consumed <sup>1</sup>	$R_s, R_t, \dots$	$n$
prices of services	$p_s, p_t, \dots$	$n$
<i>New Capital:</i>		
Quantity manufactured	$R_h, R_k, \dots$	$n'$
with cost of production	$\Pi_h, \Pi_k, \dots$	$n'$
Total excess of income over consumption, expressed in numerical terms <sup>2</sup>	$E$	1

<sup>1</sup> Thence the quantities  $Q_s - R_s, Q_t - R_t \dots$  are devoted to the manufacture of new capital and of final products.

<sup>2</sup> This excess serves for the manufacture of new capital and the constitution of new working capital.

<sup>3</sup> In my elementary treatise, "Principi di economia politica" (Biblioteca del *Giornale degli economisti*), I used the conception of utility, because it seemed to me the simplest and clearest method to explain to the beginner some of the most notable results of the new theories. This treatise will be referred to in future by the short name "Principi".

There are altogether  $3m + 2n + 2n'$  unknowns.

The question now is to see if there is an equal number of independent equations.

7. *Equations expressing the R's and E as Functions of Prices.* – Let us begin with individual budgets. It is convenient to suppose – it is a simple book-keeping artifice, so to speak – that each individual sells the services of all his capital and re-purchases afterwards the part he consumes directly. For example, A, for eight hours of work of a particular kind which he supplies, receives a certain remuneration at an hourly rate. It is a matter of indifference whether we enter A's receipts as the proceeds of eight hours' labour, or as the proceeds of twenty-four hours' labour less expenditure of sixteen hours consumed by leisure. The latter method helps to make easier the comprehension of certain maxims of which we shall speak later. Naturally we shall not use this artifice, when (§ 22) we deal with the case of services being monopolized by an individual or a group.

The individual then, selling at prices  $p_s, p_t \dots$  the quantities  $q_s, q_t \dots$  of the services of capital of which he disposes, devotes the proceeds to certain products  $r_a, r_b \dots$  and certain services  $r_s, r_t \dots$  which he consumes, saving  $e$ .

The individual, then, within the limits of the equation

$$r_a + p_b r_b + \dots + p_s r_s + p_t r_t + \dots + e = p_s q_s + p_t q_t + \dots,$$

which the economic society in which he lives imposes, after having sold all his services, reserves a part of his receipts for saving.

We shall not inquire into the criteria on which this distribution is made. It is a *fact*, and here we confine ourselves to formulating it: and to showing that if the series of prices were different, he would demand final products and consumable services in different amounts and would save a different amount.

Hence each of these quantities demanded (and likewise the amount of the individual's savings) depend on the entire series of prices, according to certain functions which it is not necessary to define here. By saying that the individual  $r$ 's and  $e$  are functions, intricate though they be, of all prices, we are only stating a *fact* of universal experience. And that is enough.

Given, then, a series of prices, the  $r$ 's and  $e$  are determinate; and consequently the  $R$ 's and  $E$  are determinate as functions of prices. Note that each one of these  $m + n + 1$  quantities is a function of *all* the  $m + n - 1$  prices of products and services.

8. *The Equations of the Equilibrium.* – Beside  $m + n + 1$ , which express the  $R$ 's and  $E$  in functions of all the prices of final goods and services, the following relationships can be established:

The first system of equations expresses the physical necessities of production: the total of the services of existing capital must suffice for final

goods and services and for the manufacture of new capital, including new working capital:

$$\text{I. } \begin{cases} Q_s = R_s + a_s R_a + b_s R_b + \dots + h_s R_h + k_s R_k + \dots \\ Q_t = R_t + a_t R_a + b_t R_b + \dots + h_t R_h + k_t R_k + \dots \end{cases}$$

There are  $n$  of these equations.

Then we have an equation, which says that the excess of incomes over consumption is used in the manufacture of new capital:

$$\text{II. } E = \Pi_h R_h + \Pi_k R_k + \dots$$

Another system of equations gives the cost of production of final goods and new capital as functions of prices of productive services:

$$\text{III. } \begin{cases} \pi_a = a_s p_s + a_t p_t + \dots & \Pi_h = h_s p_s + h_t p_t + \dots \\ \pi_b = b_s p_s + b_t p_t + \dots & \Pi_k = k_s p_s + k_t p_t + \dots \\ \dots & \dots \end{cases}$$

They are  $m + n'$  in number.

Lastly, another system expresses one of the characteristics of free competition that the price of final products and of services of new capital equal their cost of production:

$$\text{IV. } \begin{cases} 1 = \pi_a & p_h = \Pi_h \cdot p_e \\ p_b = \pi_b & p_k = \Pi_k \cdot p_e \\ \dots & \dots \end{cases}$$

There are  $m + n' - 1$  of these equations, because among the varieties of new capital is new working capital, the price of which is  $p_e$ .

For new capital the condition of the price of the services being equal to the cost of production means that the net rate of yield of new capital is equal everywhere to the interest  $p_e$  on free capital (included among the  $p$ 's of the various services).

9. Counting the number of the equations of the four systems and adding the  $m + n + 1$  relations which express the  $R$ 's and  $E$  in functions of all prices, we find in all  $3m + 2n + 2n' + 1$  equations. These exceed by 1 the number of the unknowns; but, as it is easy to see, one of the equations is the result of the others. In fact, summing up on the one hand the equalities of the individual formulae, we arrive at

$$R_a + p_b R_b + \dots + p_s R_s + p_t R_t + \dots + E = p_s Q_s + p_t Q_t + \dots,$$

which is the same result as is obtained by adding together, on the other hand, those of system (I) after having multiplied by  $p_s, p_t \dots$  and taking account of (II), (III) and (IV).

Thus we have the same number of equations as of unknowns. The entire economic system is thus determinate.

10. We have considered the technical coefficients as given quantities; now let us determine them. Some are constants; others are variables and related to each other and to the quantity produced by certain relations. These relations are those of increasing or decreasing returns, as is shown by experience. This *economic variability* of the technical coefficients is related to phenomena of the greatest importance. On these matters Vilfredo Pareto has made a most useful contribution to our science. To proceed gradually, let us begin by considering the limiting case of free competition, when, that is, the profits of enterprise are absent, and production is in the hands of one or more entrepreneurs, whose firms are similar to each other, and who are producing at the same cost. It is easy to see by what relations the technical coefficients and the sizes of the firms are determined.

To give the problem its most general solution, let us suppose that between the  $n$  technical coefficients of the product  $B$  there are  $k$  relations ( $k < n$ ) of the form:

$$f_{\theta}(b_s b_t \dots Q_b) = 0 \dots \theta = 1 \dots k$$

$n - k + 1$  equations are necessary to determine the  $n$  coefficients and the quantity  $Q_b$ . And these precisely we have, giving the minimum  $\pi_b = b_s p_s + b_t p_t + \dots$ , in which the prices are considered as constant and  $b$  and  $Q_b$  related by  $f_{\theta}$ . Thus is constituted the well-known theory of maximum and minimum relations.

11. Now let us consider, taking a step towards the real case, several competitive enterprises and their profits.

Profit, in which there is an element in addition to the wages of management, i.e. there is a differential gain, appears as soon as the competing entrepreneurs are *not* manufacturing under the same conditions. For it is evident – in the realistic case – that it is necessary to admit that, besides the technical relations between the technical coefficients, there are, for each entrepreneur, special *economic* relations, which are usually based either on the want of ability to discern and to put into action a plan which combines the technical coefficients to the greatest economic advantage, or on the impossibility of arranging that combination of maximum advantage because of the limitation on the available supply of some factor. Hence originates the *transitory* profit of various enterprises, even in static conditions.

It is easy to see how even in this case the problem may be determined. It is a question of  $\alpha$  competing entrepreneurs. There are  $\alpha$  new unknowns representing the respective individual profits  $g_1, g_2 \dots g_{\alpha}$ , and  $\alpha$  new unknowns representing the respective quantities produced.

Now in this case each entrepreneur, in organizing his production in a manner to obtain the maximum profit  $Q_b(p_b - \pi_b)$ , will consider as constants (because he is not able to change them himself) the prices of the product and of the services, and as variables the quantities to be produced and the technical coefficients. These are the conditions in which the quantity produced and the technical coefficients for each firm are determined. The profits per unit are:

$$p_b = b'_s p_s + b'_t p_t + \dots + g_1 = b''_s p_s + b''_t p_t + \dots + g_2 = \dots^4$$

If a marginal producer  $\alpha$  makes no profit,  $g_\alpha = 0$ .

But, reserving for later discussion the profits of the various enterprises, let us confine ourselves now to the limiting case to which free competition tends, in which there are one or more competing entrepreneurs who make no profit and who produce at the same cost.

12. *The 'Maximum' of Free Competition.* – The system of equilibrium equations which we have just seen can be simplified in the following manner:

the system of  $R$ 's and  $E$  in functions of prices;

the system (I) which expresses the physical necessities of production and which, obviously, will be found in any other economic régime;

the following system (II bis):

$$(II \text{ bis}) \quad \begin{cases} 1 = a_s p_s + a_t p_t + \dots & p_h = p_e (h_s p_s + h_t p_t + \dots) \\ p_b = b_s p_s + b_t p_t + \dots & p_k = p_e (k_s p_s + k_t p_t + \dots) \\ \dots\dots\dots & \dots\dots\dots \end{cases}$$

which is characteristic of free competition;

finally the system in which the technical coefficients are determined in such a manner that the costs of production may be at a minimum; and this case also, as that of the price being equal to the cost, is characteristic of free competition.

13. A noteworthy property of this equilibrium is that the partial differential of

$$\Phi = R_a + p_b R_b + \dots + p_s R_s + p_t R_t + \dots + E$$

is zero when prices are considered as constants.

The quantity  $\Phi$  can also be put in the form

$$\Phi = R_a + p_b R_b + \dots + p_s R_s + p_t R_t + \dots + \frac{1}{p_e} (p_h R_h + p_k R_k + \dots)$$

<sup>4</sup> The reader will find a graphic illustration of equilibrium, taking account of the profits of the undertakings, in "Principi", §§ 8-13.

We will show first that the partial differential of  $\Phi$ , taking prices as constants, is zero; afterwards we will interpret the economic significance of it.

In fact

(a) Leaving fixed all the other quantities  $R$ , suppose an increase in the quantity of one of the products, say  $B$ , of  $\Delta R_b$ , allowing for the services required. Then we have in  $\Phi$  on one side the increment of  $p_b \Delta R_b$ , and on the other the decrease  $(p_s b_s + p_l b_l + \dots) \Delta R_b$ ; and therefore  $\Delta \Phi$  is nil because  $p_b = p_s b_s + p_l b_l + \dots$ .

(b) Leaving fixed all the other quantities  $R$ , suppose an increase in new capital of some kind, of  $\Delta R_h$  allowing for the necessary services. Then we have in  $\Phi$  on the one side the increment  $\frac{1}{p_e} p_h \Delta R_h$  and on the other the decrease  $(p_s h_s + p_l h_l + \dots) \Delta R_h$ ; and therefore  $\Delta \Phi$  is nil, because  $p_h = p_e (p_s h_s + p_l h_l + \dots)$ .

(c) Leaving fixed all the other quantities  $R$ , suppose that in the manufacture of  $B$  there are used more of  $S$  and less of  $T$  ( $b_s$  and  $b_t$  are independent), adding or subtracting the services consumable by them. Then the variation of  $\Phi$  will be  $(p_s \Delta b_s + p_t \Delta b_t) R_b$ . But this variation is zero, because the technical coefficients were determined with the condition of  $\pi_b$  minimum.

Consequently, precisely by virtue of the conditions which are characteristic of free competition (that is, the cost of production equals the prices and the costs of production are at a minimum) given the quantity of services available, the partial differential of  $\Phi$  *when prices are considered constant* is zero.

Of this proposition we may give further demonstration.

If that equilibrium is changed in any manner whatever (for example, by changing the technical coefficients so that the costs of production are no longer the minimum; or by disturbing the equality of prices to the costs of production) so that the  $R$ 's and  $P$ 's are changed, since always, according to the individual equations, there must be

$$R_a + p_b R_b + \dots + p_s R_s + p_l R_l + \dots + E = p_s Q_s + p_l Q_l + \dots,$$

the total variation of the first section will be composed of two parts. The former is that  $\Delta \Phi$ , just now considered by us, which is obtained by differentiating with the  $p$ 's *regarded as constant* and the  $R$ 's as variables. The second, on the other hand, is obtained by differentiating with the  $R$ 's regarded as constants and the  $p$ 's as variables. It is easy to see immediately that the first part, our  $\Delta \Phi$ , is zero if in the equilibrium the equations (IV) hold. It is enough to multiply (I) by  $\Delta p_s$ ,  $\Delta p_l$ ,  $\dots$  and to sum up.

Note that this partial differential  $\Delta \Phi$ , just now considered, can be put [as it is easy to verify, finding the total differential and taking account of equa-



tions (III)] in the form  $\Sigma R(\Delta\pi - \Delta p)$ , which expression is zero if the costs of production are minimum and prices equal costs and it becomes *negative*, as might be expected, if, on the other hand, one or more prices become higher than the respective minimum costs of production.

14. Let us remember now that  $\Phi$  is the sum of all individual quantities analogous to

$$\phi = r_a + p_b r_b + \dots + p_s r_s + p_t r_t + \dots + e,$$

which we have seen in the individual equations.

Let us remember, moreover, that if the individual  $A$ , by an alteration in the economic equilibrium, obtains a positive  $\Delta\phi$ , *considering prices constant*, his situation *is improved*. *Vice versa*, his situation deteriorates if a negative  $\Delta\phi$  results. Let us demonstrate this, specifying the significance of that *improvement and deterioration*. Then let us suppose that prices vary and therefore the different  $r$ 's of the individuals vary. In the individual equations which express the usual relations the total variation of the first section is composed of two parts: the first is our  $\Delta\phi$ , *considering prices constant*, the second, on the other hand, is obtained by differentiating with prices as variables and the  $r$ 's as constants. Then

$$\Delta\phi = q_s \Delta p_s + q_t \Delta p_t + \dots - (r_b \Delta p_b + \dots + r_s \Delta p_s + r_t \Delta p_t + \dots).$$

When this  $\Delta\phi$  is positive, that is to say, if the individual holds his consumption unchanged at the new prices, he will have an excess of income over expenses. Therefore, however the individual disposes of this excess in new consumption, and independently of whatever criterion is the basis of this distribution, his situation will be improved, because even if he spends *all* the increase on a *single item* of consumption, taking all the others as at first, he will now achieve a more advantageous combination than before, improving his situation in a sense which cannot give rise to equivocation.

As for a negative change, this necessarily constrains the individual to adopt a combination  $\beta$ , less advantageous than the former combination  $\alpha$ , since if it did not, it would mean that in passing from  $\beta$  to  $\alpha$  with a positive  $\Delta\phi$ , the individual would not obtain a more advantageous combination; we have already seen that, in this latter case, a more advantageous combination *is* obtained. From these premises we come to a most important conclusion.

This conclusion is, that if in any way whatever the conditions alter, the costs of production falling and the prices remaining equal to costs,  $\Delta\Phi$  will be negative, that is to say, the individual  $\Delta\phi$ 's will either be all negative (i.e. every individual will suffer loss) or there will be some positive and others negative, the negative preponderating. That is to say, some individuals will

be benefited, others will suffer loss; the loss to the latter will be decidedly greater than the advantage to the former, in the sense that even taking all their gain from those who have gained in the change (which takes them back to their former condition) and giving it to those who have lost by it, the latter, even with such an addition, remain in a worse situation than originally: or indeed, what comes to the same thing, some of the latter with such an addition might be brought back to their former situation, but all of them certainly could not.

To explain more clearly this conception, which is of great importance, let us think of three individuals only. When that equilibrium is attained at which  $\Delta\phi_1 + \Delta\phi_2 + \Delta\phi_3$  is zero, it is implicit that every divergence from the equilibrium conditions expressing the minimum costs of production and the equality of prices to costs renders that sum ( $\Delta\phi_1 + \Delta\phi_2 + \Delta\phi_3$ ) negative. If all three terms are negative the positions of all the three individuals will become worse. If some are positive and some negative – e.g.  $\Delta\phi_1 > 0$ ,  $\Delta\phi_2 > 0$ ,  $\Delta\phi_3 < 0$  – while in absolute value  $\Delta\phi_3 > \Delta\phi_1 + \Delta\phi_2$ , if the gain of individuals 1 and 2 were transferred to 3 (who has lost) the latter would still be left with less than he had formerly.

15. One can say then, with regard to this maximum, that production organized with the two conditions characteristic of free competition does not itself maximise, as it is often erroneously said, the *sum of the products* which are afterwards distributed among the group by the competitive system. If we may be allowed for the moment to use that incorrect expression and unscientific concept “the sum of the products” (which is greater, the “sum” of a hundred litres of grain and ten of wine or that of ninety of grain and fifteen of wine?) it is not at all true that this sum of the products is maximized, because if, e.g. the individuals would be satisfied with less leisure the “sum of the products” could be increased. If the use of the word “sum” is tolerated, the only “sum” which is maximized is that of products and services, including leisure.

Nor is it correct to say that free competition leads to this maximum because within the limits of the equation

$$r_a + p_b r_b + \dots + p_s r_s + p_l r_l + \dots + e = q_s p_s + q_l p_l + \dots$$

each individual is free, with the services which he supplies, to make that choice between consumption of products, consumption of services and saving, which pleases him best; because obviously, in other régimes, although the expression and form of that equation might be different it is perfectly conceivable that the individual may be left free within the same limits to make whatever choice he pleases between consumption of products and consumption of services and saving.

And lastly, the maximum of free competition certainly does not imply that, in such a régime, every individual, with the services at his disposal, obtains a higher scale of choice than that which is possible in any other régime.

It is quite incorrect to suppose that this maximum has any such implications.

16. The maximum, we repeat, simply means this: that by substituting other conditions for one or more of the characteristics of free competition (minimum costs of production, equality of prices and costs of production) the conditions of *all* could not be improved. On the contrary, if some are benefited by this substitution their gain is less than the loss of those who suffered. So that if all their gain is taken from those who gained by the substitution, and is given to those who suffered loss by it, the latter could never retrieve their former position and some would always remain losers.

17. Such is the significance of the maximum, from which we deduce these corollaries:

(1) That each substitution of other conditions, for one or more of the characteristic conditions of free competition, is a *destruction of wealth*, in the sense that wealth which could have been produced with the available resources is not obtained.

(2) That if it is considered desirable to benefit some at the expense of others, it is much better – rather than by altering the conditions of free competition to obtain such a result *indirectly* – to make direct transfers from the latter to the former, because by such a method the harm inflicted on the latter is less, in proportion to the gain made by the former. Naturally, this is true only so far as this method of direct transfer does not noticeably alter the conditions of production.

The old economists had a *vague idea* of all this; but they had not a precise conception, nor were they able to give a rigorous demonstration. Consequently sometimes by clumsy arguments (which have a curious effect on those who are used to most rigorous logic) they arrive at conclusions which in the main are correct. These conclusions they had in fact perceived by intuition, though they believed they had demonstrated them. To have defined precisely this fundamental conception, to which we shall often refer later, and to have given it a thorough demonstration is the great merit of using mathematical analysis in political economy.

18. Before passing to monopolies and cartels, let us illustrate the genesis and the significance of a more or less graphical method, of which we shall sometimes make use later. It is a quick way, useful for obtaining immediately – provided it is adopted with due caution – a rough idea of certain results, which it would be much more laborious to deduce by using directly the system of equations of equilibrium.

For product  $B$ , for example, we have seen (7) that the price is a function not only of  $R_b$ , but of all the  $R$ 's; as, *vice versa*, the quantity  $R_b$  is a function not only of  $p_b$ , but of all the  $p$ 's. Hence it is not possible to imagine any cause whatever which makes one  $p$  vary without altering all the others and all the  $R$ 's, sooner or later, according to the friction, as we say, which the economic system presents to the propagation of these movements; even without altering the technical coefficients which, by their economic variability, are bound up in the entire system.

But it is possible to imagine an intermediate period between one equilibrium and another, in which  $p_b$  alone varies, with the consequential changes of the  $R$ 's, without the movement of variation being transmitted by  $p_b$  to all the other prices. Then for the small variations of  $p_b$  we could hold

$$dR_b = \frac{\partial R_b}{\partial p_b} dp_b.$$

That partial derivative is generally negative, as experience shows. Whence arises the conception of a small movement along *the smooth curve of demand* on either side of the position of equilibrium.

In this intermediate period, since the equation

$$R_a + p_b R_b + \dots p_s R_s + p_t R_t + \dots + E = p_s Q_s + p_t Q_t + \dots$$

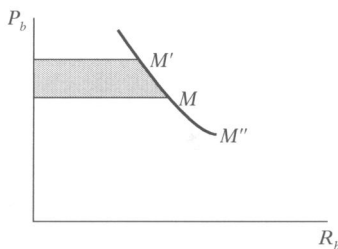
must always hold good, the usual variation of  $\Phi$ , the single price  $p_b$  being varied, will be

$$dR_a + p_b dR_b + \dots p_s dR_s + p_t dR_t + \dots + dE = -R_b dp_b.$$

This means that after the variation of the single  $p_b$  and before the variation is transmitted to the other prices; the mass of individuals has experienced a change, as if the sum of all the productive services  $Q_s p_s + Q_t p_t + \dots$  had undergone a variation  $-R_b dp_b$ ; which, apart from the second order of small quantities, is the shaded area shown in Fig. 1. Thence is derived the concept of the *variation of the consumers' surplus*. This variation gives in an approximate way, for small oscillations around the position of equilibrium  $M$ , an idea of the variation of the state of the individuals: how much they are affected by the variations of a single price  $p_b$ . This is subject to the hypothesis that this variation of one price has not so far been transmitted to other prices.

This procedure is adopted with the same caution with which, in infinitesimal calculus, one makes use of certain graphical *illustrations*, as distinct from graphical *proofs*; just as in the formula for the radius of curvature of a smooth curve it is said that it is equal to the infinitesimal length of the arc divided by the angle which the two tangents at the extremities of the same

FIGURE 1



arc make, without taking into consideration the known infinitesimal curvilinear triangle.

19. *Monopolies and Cartels.* – Equilibrium in an individualist régime exists in a medley of free competition, monopolies and cartels.

We note that in the equilibrium previously studied, which represented the full régime of free competition, each individual in the market, either as a consumer or as a producer, or as an entrepreneur, acts pursuant to the maximization of his own gain but *subject to* the market prices of products and services. He is subject to them in the sense that, as he cannot influence them in any appreciable manner by increasing or restricting the demands or offers which he makes, he will consider such prices as given constants (11). On the other hand, monopolies and cartels are characterized precisely by the fact that by increasing or decreasing supplies they can noticeably influence the prices. They therefore take account of the variability of these prices and of the influence they can exercise directly in order to increase their own profits.

20. The Monopolies which are most interesting are those of a single entrepreneur manufacturing a product and a single seller of a productive service.

Let us repeat that the origin of the difference between equilibrium in this case and equilibrium in the preceding case is that in the case of free competition the manufacturer of a product or the seller of a service cannot, by decreasing or increasing the quantity of the product or service, influence in a noticeable manner the total supply in the market, and therefore he cannot directly influence the price, which he must consider as constant. In the case of monopoly, on the other hand, by changing  $R$  the respective  $p$  can be influenced; and therefore in solving his own problem of maximizing his gains a supplier will consider this price as a variable function of the quantity he supplies and will therefore adjust the quantity to his own advantage. Now we will proceed.

21. Let us suppose the manufacture of product  $B$  to be monopolized. The entrepreneur seeks to maximize the profit  $(p_b - \pi_b)R_b$  from his monopoly. If,

as is the most general case, he can act only on the selling price of the product and not at all on the cost of production (because he is obliged to accept the prices of services as they are and cannot influence them directly, because he finds himself demanding services in competition with the manufacturers of *other* products), then, to obtain his maximum profit, he must consider  $p_b$  and  $R_b$  as variables (the latter as an independent variable) and  $\pi_b$  as a constant. Then the condition of this maximum is

$$p_b - \pi_b + R_b \frac{\partial p_b}{\partial R_b} = 0 \quad \text{or} \quad p_b + R_b \frac{\partial p_b}{\partial R_b} = \pi_b \quad (\alpha)$$

which in system (IV) is substituted for  $p_b = \pi_b$ .

22. Let us suppose one of the services, say  $S$ , is monopolized. Then the quantity at the disposal of the market is no longer *given*: it is a new unknown, which it is in the power of the monopolist to augment or diminish. At the same time there is added to the system of equations one which formulates that the sole possessor of such a productive service will try to obtain the maximum  $Q_s p_s$ ; this equation is

$$p_s + Q_s \frac{\partial p_s}{\partial Q_s} = 0 \dots \quad (\beta)$$

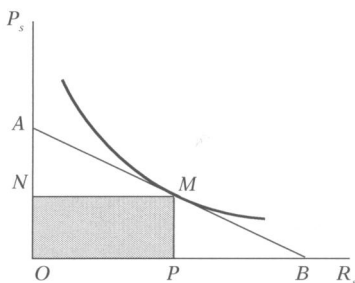
If instead of a single monopolist there is a cartel, that is a syndicate of  $\theta$  individuals, the possessors of a service which, to their own advantage they can monopolize, the preceding equation is used for the determination of  $Q_s$ , the new unknown, and in the expression of the individual relationship the  $q_s$  of each individual is determined in the second term, by the way in which  $Q_s$  is distributed between them (i.e. how each individual contributes to the total  $Q_s$  of the cartel).

Consequently also in these cases the equilibrium is perfectly determinate. It is not true that the cartelization renders the problem of price and quantity indeterminable. Given any particular agreement among members of the cartel on the distribution of the individual contributions to the total  $Q_s$  supplied to the market, and on the distribution of receipts, the entire equilibrium is determinate. But whatever may be this division of  $Q_s$  into individual contributions and this division of  $Q_s p_s$  between the members, it is obviously always advantageous to all that  $Q_s$  shall be such that  $Q_s p_s$  is maximized.

23. Our analysis of the complications introduced by cartels and monopolies can be illustrated graphically.

Let us look at the case of the cartel (Fig. 2). The quantity  $R_s$  is a function, as we know, of all prices. But if all the prices except  $p_s$  are considered constant (and the syndicate will consider them as such in aiming at its maximum profit) the relations between  $R_s$  and  $p_s$  can be represented by a smooth curve (18).

FIGURE 2

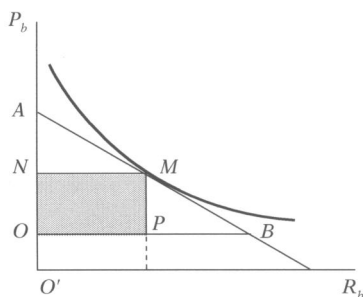


The point  $M$  of the equilibrium of the cartel (we will call it Cournot's point) is that in which the shaded rectangle is maximized; it has the property that the projection  $PB$  is equal to the abscissa  $OP$ , also  $AN = NO$ .

And, since  $OP$  is the  $Q_s$ ,  $p_s + Q_s \frac{\partial p_s}{\partial Q_s} = 0$ , therefore  $NO = p_s$  and  $AN = -Q_s \frac{\partial p_s}{\partial Q_s}$ .

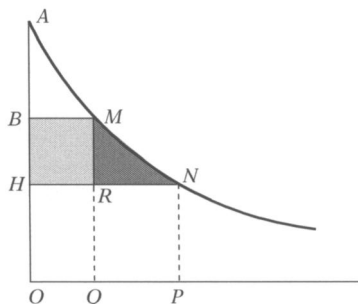
Let us now look at the case of monopoly (Fig. 3). The problem is to maximize the shaded rectangle ( $OO'$  is the cost of production). It is maximized when  $AN = NO$ ; or when  $-R_b \frac{\partial p_b}{\partial R_b} = p_b - \pi_b$ .

FIGURE 3



24. As we said so much about it in section 17, there is no need for another demonstration of the proposition that monopolies and syndicates create a difference from the equilibrium of free competition which may be described as a destruction of wealth, in the sense that if some (the monopolists) obtain a profit by it, others (the consumers) lose more. The latter would lose less if, without altering the conditions of productions of free

FIGURE 4



competition, they surrendered freely to the former that increase of wealth which the constitution of monopolies and syndicates would have procured for those people.

Using (as in Section 18) the crude graphical representation, we note that precisely the same conclusion is revealed (Fig. 4). Indeed, in passing from the point  $N$  (free competition, price equal to cost) to point  $M$  (monopoly, with the condition of  $BMRH$  maximized) the loss of some is  $BMNH$  and the gain of the monopolist is  $BMRH$ : the loss of the former, then, is greater than the gain of the monopolist by  $MRN$ . There would have been less disadvantage to all if  $BMRH$  had been taken away directly and been given freely to the monopolist, leaving production as before: the destruction of  $MRN$  would have been avoided<sup>5</sup>.

24A. *Money*. – Economic equilibrium is the starting-point for all further inquiry. Consideration of as many other problems as we please naturally rise from that point, as branches from the trunk of a tree.

Let us take an example: money.

In order to see things with a greater clarity, let us suppose – a temporary hypothesis which we will modify immediately – that the merchandize  $A$ , instead of *money*, be the *numeraire* (that is, that in terms of which the prices are expressed) and that one of the productive resources,  $M$ , already included in the equilibrium, is *money*, i.e. it has that special function which, in production and exchange, it fulfils independently of its numerical quality in the sense now defined. Individuals and entrepreneurs will not require a quantity,  $R_m$ , of money, but a certain quantity,  $R_m \Pi_m$  ( $\Pi_m$  is the price of money expressed in the *numeraire*) which is a function of all the prices. For this money-good, as for everything else, the quantity,  $R_m$ , the  $\Pi_m$  and  $p_m$  (the price for the use of it), will be determined in the equilibrium. Likewise there will

<sup>5</sup> See "Principi", §§ 16-18.



be a definite quantity,  $R_a$ , of  $A$ , which is both a commodity and the *numeraire*. All is determinate.

Now let us reject that temporary hypothesis and identify  $M$  with  $A$ , in the system, making  $A$  become not only a commodity and the *numeraire* but also money. It is easy to see that even now the problem is entirely determinate. Indeed, in the system of equations of the equilibrium we have only to introduce these variations:

(1) In the place of  $m_s, m_i \dots$  write  $a_s, a_i \dots$

(2) In the place of  $R_m$  write  $R_{am}$  understanding this to be the quantity of  $A$  money, to distinguish it from the quantity of  $R_a$  goods.

(3) To introduce the new equation  $\Pi_m = \pi_a$ . But it is easy to see there is another way. Indeed, of the three

$$\Pi_m = a_s p_s + a_i p_i + \dots$$

$$\pi_a = a_s p_s + a_i p_i + \dots$$

$$\Pi_m = \pi_a$$

one is the consequence of the other two.

The problem of the monetary equilibrium, then, is determinate. The quantity of  $A$  goods is given here as  $R_a$ , and the quantity of  $A$  money,  $R_{am}$ . The equilibrium is stable, and in dynamic changes the equalization of the prices of  $A$  goods and  $A$  money (both the prices equal to 1) is maintained by shifting the commodity to or from the monetary use. That is, of course, in a closed market.

25. The reader will notice that all this theory of the economic equilibrium, in which we have compressed into a system of equations many varied circumstances, of which we take account at one moment<sup>6</sup> – all this theory, we say, we have expounded without it being necessary to refer to any concept of *utility*, the *final degree of utility* or to effort-costs and such-like. “Synthetic economics” can do without all that, without a single one of its theories suffering for it. It does not need to avail itself of any other concepts but those old, well-known and clear ones of demand, supply and cost of production expressed in a *numeraire*, and not in terms of vague “efforts and sacrifices”.

This is the best proof that there is no necessary bond between the new theories of “synthetic economics” and the theories of the final degree of utility.

It is useless to make out that economics would not have been able to attain the degree of *synthesis* of the new theories unless mathematical *analysis* had been previously applied to it.

And now we pass to the Collectivist Régime.

<sup>6</sup> Since this is what the recently developed doctrines amount to, it seems more appropriate to describe them as “synthetic economics” than as “mathematical economics”.

### III. THE COLLECTIVIST RÉGIME

26. *The Statement of the Problem.* – Some resources remain the property of the individuals (e.g. that which they devote to personal uses): let them be  $M, N \dots$  to  $l$  terms. Let the resources which become the collective property of the State (e.g. fixed capital and land capital) be  $S, T \dots$  to  $n - l$  terms.

The Ministry of Production has to solve the problem of combining these individual and collective services in order to procure the *maximum welfare* for its people. We shall see in what precise sense this vague formula can be understood. The Ministry has studied the very complex problem and has solved it, on the basis of a certain formula of distribution which has been established by the community, on certain ethical and social criteria, with which we do not propose to concern ourselves directly. Such a formula of distribution we suppose (we shall deal with the wherefore later) may be embodied in a certain law, according to which is distributed between the members of the community, what in the old régime was the *yield* of resources now appropriated by the State and what was the profit from various enterprises now administered directly by the State (i.e. socialized). We shall see later whether *all* this *income* can be effectively distributed among the community.

27. If the exposition of the solution of the problem were to follow step by step the route followed in the inquiry, it would be long and confusing.

Therefore, with a view to brevity and clarity of exposition, we shall first enunciate the conditions in which the Ministry is faced with the task of solving the problem. We shall see how in such conditions, and with the criterion of the maximum collective welfare, it succeeds in determining the equilibrium perfectly, with as many equations as unknowns.

*Later* we shall return to the conditions which it has imposed on itself and we shall show how, if the conditions were different, scientific collectivism would break down either because the problem was indeterminate (the number of conditions insufficient to determine the equilibrium), or because the problem is not only practically but also logically insoluble (the number of equations exceeding the number of quantities to be determined), or, indeed, even when the number of conditions equals the unknowns and the equilibrium is therefore determinate, because the maximum of collective welfare obtainable in this equilibrium would be less than that necessary to provide the distribution formulated.

Hence it is preferable for it to plan production in its own way, and if it still wishes to correct the distribution it should work directly on the formula of distribution, varying certain coefficients  $\gamma$  which we shall define later, rather than directing production on lines inconsistent with the fundamentals of its own arrangement.

Hence the reader must expect that the conditions which we have posited here will be discussed later (§§ 39-54), after the solution of the problem, when a comparison will be made between those conditions and others which could have been posited.

28. Here are the conditions in which the Ministry of Production faces the problem:

(1) There is no money. There are *products* of a certain work of a given kind. There are no *prices*: but the Ministry maintains, for no other purpose than the social accounts, some method of determining ratios of *equivalence* between the various services and between the various products and between products and services.

(2) On the basis of these equivalents<sup>7</sup> the individuals themselves bring their *products* to the *socialized shops* to obtain consumable goods or to obtain from the social administration permission to use some resources of which the State is the proprietor.

The Ministry also maintains ratios of equivalence between the services of socialized resources and other goods, because it is agreed (we shall state the reasons later) that it would be a grave social loss to cancel arbitrarily the equivalences of these socialized resources. Let  $1, \lambda_p \dots \lambda_m, \lambda_n \dots \lambda_s, \lambda_t \dots$  be the equivalences determined upon.

29. (3) The members of the community can enjoy the benefit of the quantity  $Q_s \lambda_s + Q_t \lambda_t + \dots$  which we will call  $X$  (remember that  $Q_s, Q_t \dots$  are the quantities of collectively-owned resources) either by an *indirect* distribution, the equivalents of the products being reduced, or by a *direct* distribution, that is giving to the members a *supplement* (to income) which is a quota of  $X$ .

The Ministry of Production has agreed that, generally, from the point of view of the greatest collective welfare, the *direct* distribution is preferable to the *indirect*.

The same cannot always be said for certain economic quantities which appear in the collectivist equilibrium and which are analogous to the *profits* of the old régime. We shall discuss them later.

30. (4) Being obliged to proceed with the system of *direct* distribution of  $X$ , the Ministry has decided, in agreement with the people, to try a certain system of distribution of  $X$  as a supplement to incomes. To each individual belongs  $\gamma X$ . The  $\gamma$  could be different for every individual or different for different groups or arranged in such other ways as are possible. We shall dis-

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<sup>7</sup> We will discuss later if and when it is convenient, in the interests of the community, to establish different equivalences for the same goods according to the various categories of the individuals.

cuss these different arrangements afterwards. For now and throughout the greater part of our discussion, let us suppose that  $y$  is determined and differs from one individual to another. It is clear that  $\Sigma y = 1$ .

31. (5) As for saving, the Ministry, although the people do not wish to hear the words "saving" and "interest", still ought to arrange so that all its productive services are not directly consumed or employed in the production of goods for consumption. Capital, or if we do not wish to speak of capital, the *means of production*, is used up and unless something is substituted for it, it will be necessary to reserve a part of the productive services for the manufacture of it.

But that is not all. The Ministry knows that if it devotes an *adequate* portion of productive services to this manufacture of the *means of production* it will in the future assure a still greater benefit to its people. The Ministry therefore requires some saving to be done. If it is left to individuals to save as much as they like (they then being obliged to lend the savings freely to the State), the amount of saving may not be sufficient to provide for the manufacture of that quantity of new capital which will be considered of maximum social advantage. It could impose a greater saving on individuals; but what if these are not content and prefer a greater present consumption to a greater one in the future? It could deduct from  $X$ , before proceeding to the direct distribution of it, that amount which it thinks appropriate for the manufacture of new capital; but it is agreed (we shall see later, in the sequel, the reason for this) that by such a method it would attain a collective maximum *less* than that which is possible by adopting the following method: let it choose at random a rate of *premium* for *deferred consumption*; let it then see how much saving on the basis of this premium its people put freely at its disposition. Then let it find out if with this sum of saving it is possible to manufacture such a quantity of new capital that it will be able, in the future, to put at the disposition of the people a quantity of products and consumable services so great that it can really give them the promised premium for deferred consumption. And by trial and error, raising and reducing the promised premium, it will eventually make its promise in terms which can be realized. By such a method it could provide for their greater future welfare without disturbing their freewill and without interfering with that distribution which each one makes of the *income* he receives for his work, between his present and future needs.

It could, if it wished – and nothing prevents it – prohibit the savers from lending their savings to others and oblige them to lend them to the State so that the production of some goods would be the monopoly of the Government. In the collectivist régime, the Ministry of Production orders the use of individual saving to be sold *only* to the Government.

32. (6) In distributing his earnings, which he receives in exchange for his services – according to the established equivalents of the Ministry – and that amount which he receives as a supplement to distribution (X), between consumption of various kinds and saving the individual is left free to choose, according to his own pleasure.

The Ministry of Production, after mature reflection, imposes these same conditions on itself in striving to provide the maximum collective welfare. Consequently it ought to order production so as to obtain the maximum benefit for its people with the services of which the State disposes and those of which the individuals dispose. These have the freedom, in ordering their own individual economies, to make the choice they believe most convenient, consistent with the equation

$$r_a + \lambda_b r_b + \dots \lambda_s r_s + \lambda_t r_t + \dots + e = \lambda_m q_m + \lambda_n q_n + \dots + \gamma X.$$

33. *The Collective Maximum.* – The Ministry of Production commences with the adoption of the technical coefficients which happen to exist at the time (but which satisfy their technical equations). It does not for the present preoccupy itself with the economic variability of these coefficients. It fixes, moreover, at random, a series of  $R$ 's which, however, accord with the physical necessities of production (that is System (I) of § 8). It is absolutely essential that, having chosen the technical coefficients, whatever afterwards may be the system of production which it wishes to follow, the quantity of productive services available must always be precisely that which is necessary to provide for services which are consumed directly and for the manufacture of products and of new capital.

Let it give now, a *random* series of equivalents and the modifications which may be necessary in order that these technical conditions of production (System (I)) may be satisfied. It is understood that there is not a *single* system of equivalents which satisfies these conditions. If it, indeed, announced at random  $m + n - 1$  equivalents of products and productive services, each of its people will make, as we say, a schedule. The individual schedules will give, for the series of equivalents selected by chance, the individual  $r$ 's and  $e$ , whence are derived the totals  $R$ 's and  $E$ . But as System (I) gives a number of relations between these  $R$ 's and the  $E$ , less than the number of equivalents, which are  $m + n - 1$ , the system of equivalents satisfying System (I) will admit an infinite number of solutions. Then the Ministry decides on one among those which satisfy System (I) as a starting-point. It will then make adjustments in such a way as to attain the end of the maximum collective welfare.

34. What concrete and unequivocal significance must be attached to this very vague expression "maximum collective welfare"?

If the Ministry corrects one of the equivalents consistently with (I), the individual will make a new choice, which will be more or less advantageous than the preceding choice according as

$$\Delta r_a + \lambda_b \Delta r_b + \dots + \lambda_s \Delta r_s + \lambda_t \Delta r_t + \dots + \Delta e$$

which we call  $\Delta\theta$ , is positive or negative (14) according to which, we will say, for the sake of brevity, the individual will be *higher* or *lower*.

The meaning of the *collective maximum* would be patent if, by successive attempts, the Ministry could arrive at such a series of equivalents that every further modification of it would place *all* individuals lower. *But such a series of equivalents does not exist; it is useless to try to find it.* It would be necessary to find such a series of equivalents, that by modifying one of them by a very small quantity, the  $\Delta\theta$  for each individual would be reduced to zero. And that is *impossible*; since, as we shall now see, the sole condition for reducing to zero not the individual  $\Delta\theta$ 's but their sum  $\Sigma\Delta\theta$ <sup>8</sup>, implies as many conditions as are sufficient to determine completely all the equivalents.

We must bear in mind the possibility that, by making use of the great freedom with which the individual  $\gamma$ 's can be varied (subject to the sole condition that  $\Sigma\gamma = 1$ ), we can obtain a series of  $\gamma$ 's and of such equivalents that not only  $\Sigma\Delta\theta$  is zero but all individual  $\Sigma\theta$ 's are zero also. We will show in an appropriate place (53) that this is impossible.

35. What does the reduction of  $\Sigma\Delta\theta$  signify? To eliminate

$$\Delta R_a + \lambda_b \Delta R_b + \dots + \lambda_s \Delta R_s + \lambda_t \Delta R_t + \dots + \Delta E$$

means that every other series of equivalents, different from that which accords with this condition, would make that sum negative. That is to say, either it causes a decline in the welfare of all or, if some decline while others are raised, the gain of the latter is less than the loss of the former. (So that, even taking all their gain from those who gained in the change, reducing them to their former position, to give it completely to those who lost, the latter would always remain in a worse situation than their preceding one, without the situation of the others being improved.) Since it is absurd to attempt to resolve the *impossible* problem of finding such a series of equivalents that every further alteration would produce a reduction of welfare for everyone, we will consider that the sole criterion of maximum welfare which the Ministry of Production can use is  $\Sigma\Delta\theta = 0$ .

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<sup>8</sup> Which does not mean eliminating every *individual*  $\Delta\theta$ , for the individuals may not be *identically* provided with the resources and have *identical* tastes.

36. *How the Equilibrium is Determined.* –  $\Sigma\Delta\theta$  can be put in the form

$$\Delta R_a + \lambda_b \Delta R_b + \dots + \lambda_s \Delta R_s + \lambda_t \Delta R_t + \dots + \Delta_h \Delta R_h + \Delta_k \Delta R_k \dots$$

calling  $\Delta_h, \Delta_k \dots$  the quantities of saving necessary for the manufacture of a unit of  $H, K \dots$

Let us remember that in the first approximate solution the Ministry of Production had assumed a series of technical coefficients *at random* (though satisfying their technical equations) and one of such possible series of equivalents and of  $R$ 's as will satisfy System (I).

Now it is necessary to correct this series of quantities so long as successive corrections always give a positive  $\Sigma\Delta\theta$ , and stop at that point at which further corrections give a zero increment, a sign that the maximum is attained and that further modifications would give rise to a decline in welfare.

37. The technical coefficients are not changed at first: this task is reserved for later.

Keeping an eye on the System (I) of the physical necessities of production which must always be satisfied:

(a)  $R_b$  is increased by  $\Delta R_b$ , the necessary services being taken from those directly consumed. Then  $\Sigma\Delta\theta$  is constituted by the increment  $\lambda_b \Delta R_b$  in the product *less* the diminution

$$(\lambda_s b_s + \lambda_t b_t + \dots) \Delta R_b,$$

in the consumable services. Therefore in these changes the Ministry ought to stop when the total increment is zero, which can never happen except when

$$\lambda_b = \lambda_s b_s + \lambda_t b_t + \dots \quad (\alpha)$$

For the purpose of verification, and because thereby the significance of this argument will appear still more clear, let us begin by considering a situation in which the equivalent of  $B$  (which is afterwards the *price*, under another name, expressed in terms of that special kind of work which is called the *goods*) is greater than the cost of production. In such a case, the Ministry of Production, in the interests of the community, agrees to increase  $R_b$  and to decrease the consumable services, because by manufacturing more of  $R_b$ , the addition being  $\Delta R_b$ , there is for  $\Sigma\Delta\theta$  on the one hand the increase  $\lambda_b \Delta R_b$  by the increase in  $B$ , and on the other hand the diminution  $(\lambda_s b_s + \lambda_t b_t + \dots) \Delta R_b$ , by the diminution in consumable services. The net result of this is evidently advantageous because, by hypothesis,  $\lambda_b$  exceeds  $\lambda_s b_s + \lambda_t b_t + \dots$ . The maximum will be achieved only when there is no more advantage to be gained by such adjustments, which is when  $\lambda_b = \lambda_s b_s + \lambda_t b_t + \dots$

(b) Increase one of the new productive resources  $H$  by  $\Delta R_h$ , taking the services necessary from those directly consumed. Then for  $\Sigma\Delta\theta$  there will be on the one hand the increase  $\Delta_h\Delta R_h$  and on the other the decrease

$$(\lambda_s h_s + \lambda_t h_t + \dots) \Delta R_h;$$

and hence, with the same reasoning as before, we arrive at the condition

$$\Delta_h = \lambda_s h_s + \lambda_t h_t + \dots \quad (\beta)$$

(c) Now let us proceed to the savings. The Ministry disposes of a quantity of saving

$$E = \Delta_h R_h + \Delta_k R_k + \dots + R_e,$$

with which it must increase as much as is possible the total quantity of services available for subsequent production. It will approach this maximum, by transferring new capital from one use to another, until,  $\lambda_h \lambda_k \dots \lambda_e$  being the equivalents of the services of the new kinds of capital<sup>9</sup>  $\lambda_h R_h + \lambda_k R_k + \dots \lambda_e R_e$  reaches the maximum.

This condition of the maximum is only satisfied, evidently, when

$$\frac{\lambda_h}{\Delta_h} = \frac{\lambda_k}{\Delta_k} = \dots = \lambda_e \quad (\gamma)$$

(d) Now we proceed to the technical coefficients. The Ministry, in the first approximate solution, had chosen them in such a way that they should simply satisfy their technical equations. But we know that some of them are variables, in the sense that some can be diminished while in others there is a compensating increase. Let  $S$  and  $T$  be the services for which in the manufacture of  $B$  these variations can be made. Then, per unit of  $B$ , more of  $S$  and less of  $T$  will be employed as far as is advantageous from the point of view of the collective maximum. The  $\Sigma\Delta\theta$  is constituted, with regard to the consumable services, by an addition  $\lambda_t R_b \Delta b_t$  and a diminution  $\lambda_s R_b \Delta b_s$ . Therefore the variation is zero if

$$\lambda_s \Delta b_s + \lambda_t \Delta b_t = 0$$

which is one of the conditions of the  $\lambda_b$  minimum when the *economic* variability of the technical coefficients is considered.

38. Taking account of what we have just said on the technical coefficients and glancing at the relations ( $\alpha$ ), ( $\beta$ ), ( $\gamma$ ) of § 37, it is immediately evident:

<sup>9</sup>  $\lambda_e$  is the *premium* for deferring for one unit of time consumption of one unit of saving.



(1) That the system is perfectly determined: there are as many equations as unknowns.

(2) That the Ministry of Production in this perfecting of its first approximate and indeterminate solution (the sole criterion of perfection being the maximum collective welfare) comes to the conclusion that production should be so organized that (with the systems of technical coefficients, of the  $\lambda$ 's and  $R$ 's) *the cost of production may be minimized* and that *the equivalents for the products and for the additions to capital may be such as will correspond to their respective costs of production*.

(3) That the system of the equations of the collectivist equilibrium is no other than that of the free competition.

Which only means that with equal resources (the quantities  $Q$ ) the economic quantities of the collectivist equilibrium ( $\lambda$ ,  $R$ , etc.) will be the same as those in the individualist equilibrium; and that is due to the presence of that *supplementary term*  $\gamma X$  in the individual equations of the collectivist régime, which does not occur in the individual equations of the individualist régime.

39. *The Distribution of X.* – Now is the time to discuss the conditions (§§ 28 to 32) which the Ministry has considered as the basis of its problem.

There are five problems concerned here: the distribution of services possessed by the State; saving and the creation of new capital; the distribution of the profits from the undertakings; multiple prices; and the supplements to income ( $X$ ).

Let us discuss them in order.

If the productive resources  $S, T \dots (n - 1$  in number) are the property of the State, there are two different ways of enabling the community to reap the benefit of this collective property: either that which we have assumed as one of the conditions in the solution of the problem of the collectivist equilibrium (that is, the *direct* division of  $X$ , giving to each individual a supplement to his income  $\gamma X$ ); or that of reducing to zero, in the cost of production, the equivalent of the services of resources which are the property of the State, and taking as the equivalent of each product (the  $\lambda$ , which is subsequently the *price*) the cost of the direct personal services which are required for its production. When the product is made with others, this cost is found by dividing the total cost in personal services by the entire quantity produced.

40. This system of indirect distribution, coupled with the reduction of the equivalents of the services of collective property to zero, is, at bottom, Marx's theory of value.

Those people who have criticized Marx have justly directed attention to the fact that such a system would be far from achieving the result, "to each person the entire product of his labour", which is asserted to be connected

therewith, because it is evident that a certain quantity of work of a given kind would be rewarded by a greater or smaller quantity of a certain product, according to the quantity and quality of the State-property with which it is employed. Hence the distribution of the product, made by such a system, is very far from realizing the formula of "the whole produce". But showing that this formula is not realized does not mean that indirect distribution is shown to be unsuitable. With more effect is it remarked that even when some resources are collective property the State can do no less than fix a price for their services, since there would otherwise be an enormous waste of these, with a consequent destruction of wealth. These services would be used in a large measure, not for further production, but as consumable services, and of those employed productively there might easily be an excess in one kind of production, which excess would be more useful socially in another industry in which there was a deficiency of resources.

This is the correct and fundamental argument against indirect distribution and in favour of direct distribution: the impossibility of obtaining a maximum as high as that which could be achieved with the latter method.

41. Of such a truth we can give, in a few words, a more general and "synthetic" demonstration which can be applied equally to all those systems which propose to reduce to zero the equivalents of all or some of the services of those resources which become collective property.

To wish that the  $n - l$  quantities  $\lambda_s, \lambda_l \dots$  may be equal to zero, is to introduce into the general system of equilibrium, which we have seen *entirely* determined,  $n - l$  new equations. Hence either there is an impossible problem (the number of equations greater than the number of unknowns), or, to make it at least logically possible, it is necessary to exclude from the system  $n - l$  of the equations which are already there. And as this exclusion cannot be done by taking the equations of the  $R$ 's from System (I), because they express the physical necessities of production which any economic order whatever must necessarily respect; then to make the problem possible, it would be necessary to exclude as many equations as those which express the minimum costs or the equality of prices and costs. This means that it is necessary to exclude as many equations from those which express the obtaining of that certain maximum; exclusions by virtue of which it certainly could only obtain a lower maximum. The Ministry of Production, instead of rising to the limit, would be forced to stop half-way.

Hence one can affirm that the better way for the Ministry of Production to provide for the welfare of its subjects, is not that of indirect distribution (i.e. the reduction to zero of the  $\lambda$ 's of the services of collective property), but that of *direct* distribution of supplements to income.

42. The collectivists persist in defending themselves, by expounding, with subtle and laborious interpretations, certain propositions which are either contrary to facts or do not bear a penetrating analysis. They do not appear to think that, if they are to remain collectivists, they must now cast off these gross errors which they derived from a nebulous vision of the phenomenon and from a muddled idea of the mutual dependence of economic quantities.

Of course their attitude in this respect is reminiscent of the reluctance with which the dogmas of a religion are discussed, especially when the latter has great propaganda value.

In addition there is a consideration of great moment in a collectivist régime: that is, that indirect distribution is rigid and does not permit certain ethical and social criteria to be observed with all that liberty which is realized (by giving opportunity values to the  $\gamma$ 's) by direct distribution.

43. *Saving and the Creation of New Capital.* – For the discussion of the condition which the Ministry has imposed on itself concerning saving and the creation of new capital it will be enough for us to make:

(1) A brief observation on what we should call the productivity of capital.

(2) A comparison between the method followed by the Ministry of Production and another which it would be possible to follow, by deducting from  $X$ , before distributing it, that part which is necessary for the manufacture of new capital. Here it will be easy to show that by this second method it would realize a lower collective maximum than that which it can secure with the system preferred.

44. As for the first point, it is necessary to understand well that whether some capital is the property of individuals or whether it is collective property, does not upset the technical fact, that by once subtracting a part of the disposable productive services from the production of consumption goods, and then to produce new capital (*new means of production*, if that term is preferred), there is secured for always an increase of production greater than the *amortization* of capital.

Let us express this conception, which is the crux of the matter, with greater precision.

With the quantities  $R_s + R'_s$  and  $R_t + R'_t$  of the services  $S$  and  $T$  it may be possible for us to manufacture the quantity  $R_b + R'_b$  of the product  $B$ . We are speaking of a given unit of time, e.g. one year. In this unit of time we may sacrifice the consumption  $R'_b$  and with the services  $R'_s$  and  $R'_t$  we may manufacture instead some capital  $R_k$ . And let us call  $\epsilon$  the fraction of  $R_k$  which it is necessary to manufacture every year in order to maintain the quantity intact (amortization).

In the next unit of time, and so in continuation, with the same services  $R_s + R'_s$  and  $R_t + R'_t$  along with  $R_k$  after having taken away from those services the part which is necessary for the reintegration of  $R_k$  we could have, instead of the product  $R_b + R'_b$  which we obtained formerly, a quantity of product which we shall call  $\bar{R}_b$ , which is obviously given by these equations:

$$\begin{aligned} R_s + R'_s &= b_s \bar{R}_b + \epsilon k_s R_k \\ R_t + R'_t &= b_t \bar{R}_b + \epsilon k_t R_k \\ R_k &= b_k \bar{R}_b. \end{aligned}$$

It often happens technically – and the most obvious experience shows it – that with the choice of an appropriate method  $\bar{R}_b > R_b + R'_b$ ; thus this is the criterion on which the decision, whether to manufacture capital or not, is based. That condition is necessary though not always sufficient. Then with the sacrifice of  $R'_b$  once made, there is an everlasting additional product  $\bar{R}_b - (R_b + R'_b)$ . Hence there is the possibility of a premium on deferred consumption of  $\frac{\bar{R}_b - (R_b + R'_b)}{R'_b}$  for every unit of  $B$  subtracted from present con-

sumption<sup>10</sup>. It is precisely this purely *objective* technical fact, which does not depend in the slightest on whether the capital is individual property or collective property, which gives the Ministry the means to promise a premium on deferred consumption to those who are willing to provide it with savings for the construction of the new means of production. In substance, these people promise not to present a part of their earnings at the general shops to obtain goods, but to deposit it (though it continues to be their property) with the Ministry. The Ministry is thereby enabled to manufacture, with the total available services, a smaller quantity of final products and to set aside a part of the same services for the manufacture of new means of production. These new means of production will then be available to it in successive periods of production. It is precisely this *objective* fact which is the origin of what may be called the *economic productivity* of savings employed in production even in the collectivist régime.

45. Now we pass to another point: is it advantageous that the Ministry of Production, instead of having recourse to individual saving and promising (in order to secure a sufficient quantity of it) a premium on deferred consumption to those individuals who supply it, should, before distributing  $X$ , deduct that part of it which is considered necessary for the creation of new capital?

The criterion is, and must be, always the same: the greatest welfare for society.

<sup>10</sup> For a very elementary illustration, see "Principi", § 37.

Let us leave aside the consideration that by the second method the Ministry would take no account of the wishes of its subjects, who might prefer a greater  $\gamma X$  to-day to a smaller future increment; and let us also leave aside the consideration that the Ministry would by such a method be without any means of determining the most advantageous quantity of new capital to create. We will confine ourselves here to viewing the case exclusively from the point of view of the collective maximum.

Then, in order to manufacture by this second method the same quantities of new capital  $R_h, R_k \dots$  the Ministry distributes to the community an amount reduced by  $E$ . But each individual, even without the promise of a premium for deferred consumption, and simply for the provision of future needs, might for his own advantage decide not to consume all his *earnings*, but to save a certain amount. Hence there is a certain sum of individual saving, which we will call  $E_i$  to distinguish it from the quantity  $E$  which the Ministry, by reducing  $X$ , uses for the manufacture of new capital.

$E_i$  is the sum of all the  $e_i$ 's which result from the individual equations, which now become like this:

$$r_a + \lambda_b r_b + \dots + \lambda_s r_s + \lambda_t r_t + \dots e_i = q_m \lambda_m + q_n \lambda_n + \dots + \gamma(X - E).$$

Or for the community:

$$R_a + \lambda_b R_b + \dots + \lambda_s R_s + \lambda_t R_t + \dots E_i + E = Q_m \lambda_m + Q_n \lambda_n + \dots + Q_s \lambda_s + Q_t \lambda_t + \dots$$

That is to say, that with this second method (i.e. the method of the Ministry deducting from  $X$  the quantity  $E$  necessary for the manufacture of new capital, before distributing  $X$  among the people) the whole body of individuals is forced to limit the sum of goods and services consumed *more* than they did with the other system, with the prospect of a future increase of products and services *no greater* than that which the other system offers. Therefore evidently, in the interests of the maximum welfare of the community, the former method is preferable to the latter.

This conclusion will be more readily understood, if it is realized that this second method (which is not to be preferred) does not use, for increasing goods and services in the future, that sum of money which various individuals still save even without the promise of a premium for deferred consumption.

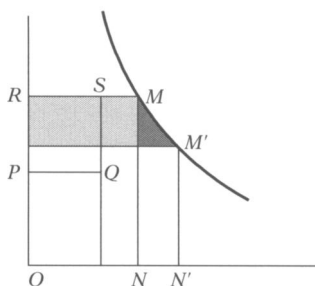
46. *The Distribution of the Profits of the Undertakings.* – The problem is in these terms: the product  $B$ , for example, is manufactured in two different ways, each with its own technical coefficients. Hence there is a *profit* for the method of production which costs less. The undertakings being socialized, this profit belongs to the community. It can be distributed among the members of the community in two ways: either *directly*, taking  $\lambda_b$  equal to the

higher cost and adding the profit  $G$  to the  $X$  which is distributed to the community; or *indirectly*, lowering the price to the *average* cost of production. Which is more advantageous?

47. Such questions we can solve by a simple graphical device.

Let the product  $B$  be manufactured in two ways at different costs, as Fig. 5 indicates. The quantity produced is  $ON$ , the higher cost  $MN$ , the profit of the lower cost undertaking  $RSQP$ . Let  $M'N'$  be the average cost, so that the obliquely shaded area will be equal to the profit  $RSQP$  of the lower cost enterprise. It is clear that at the average cost the consumption will be  $ON'$ . And it is also clear that if in passing from the production of  $ON$  to  $ON'$  – we will say in passing from  $M$  to  $M'$  – the average cost remains the same, the lowering of the price is preferable to the direct distribution of the profit, because with the latter method the community gains the shaded area, while with the method of reducing the price all the shaded area *plus* the black area is gained. Such a conclusion is true *a fortiori*, if in passing from  $M$  to  $M'$  the average cost diminishes.

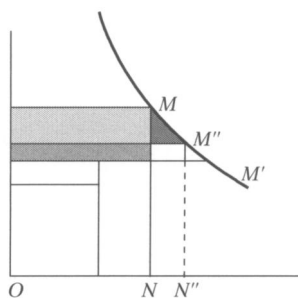
FIGURE 5



But if, instead, the average cost increases, because the new consumption  $NN'$  has to be produced at a higher cost than the two preceding ones or at a greater new cost, then according to the position, either the direct or indirect distribution of the profit is preferable.

In Fig. 6  $M'$  is the level of the previous average cost when the production is  $ON$ . The profit is shown by the horizontally and vertically shaded areas combined.  $M''$  is the level of the new average cost when the production becomes  $ON''$ . Let us call the two shaded areas  $a$  and  $b$  respectively ( $a$  the horizontally shaded and  $b$  the vertically shaded) and the black area is  $c$ . It is clear that with the system of direct distribution of the profit the community gains  $a + b$ . With the system of indirect distribution, that is, with the lowering of the price to the average cost, it gains  $a + c$ . Hence the first or second

FIGURE 6



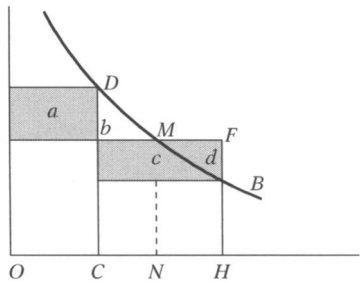
method will be the more advantageous according as  $b$  is greater than  $c$  or *vice versa*.

48. We have said this because such an aspect of the question cannot be avoided in our analysis.

However, as experience shows, the total sum of the profits is in reality unlikely to be large (there are losses as well as profits); and it will still be necessary to use a part of these profits as remuneration for the work of those people who, as assistants of the Ministry, are engaged in endeavouring to keep the cost of production as low as possible; and lastly, as we have already noticed, every method of indirect distribution implies a loss of freedom - curtailing the liberty of giving to the  $\gamma$ 's the most advantageous values from ethical and social aspects. For these reasons the Ministry would decide that there was no case for departing from the general principle of *direct* distribution, even in the sphere of profits. This decision would be reinforced by the fact that any such departure would give rise to further practical complications, and the Ministry would, apart from this, as we shall see later, already find itself in the midst of a multitude of complications arising out of the *practical* resolution of the equations of the equilibrium.

49. *Multiple Prices.* - The consumption of the product  $B$ , for example, may be  $ON$ , with the price  $MN$  which is equal to the cost of production (Fig. 7). To extend the consumption of the product and to render it more widely accessible, we can increase the price of a part of the supply, in order to lower the price of the other part, making the adjustment by a redistribution of the total cost: for example, the amount  $OC$  could bear the price  $CD$ , while for  $CH$  the price would be  $HB$ . If when the output is increased the cost of production does not vary (then the two shaded rectangles are equal) it is obvious that this proceeding implies a destruction of wealth; for it is better to take directly from some to give to others; or it is better to work on the  $\gamma$ 's.

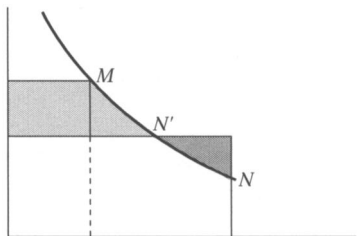
FIGURE 7



In fact, passing from the position of equilibrium with a single price to that with a double price, in this case of costs remaining constant when output increases, a destruction of wealth,  $MBF$ , is caused. This may be easily verified by considering what, in passing from one state to the other, will be the variation of the consumers' surplus: as a result of the change it decreases by  $a + b$  for one part and increases by  $b + c$  for the other, a net increase of  $c - a$ ; but  $a = c + d$  because the average cost per unit is  $HF$ ; therefore, on the whole, the consumers' surplus is diminished by  $d$ .

50. In order that there shall be no loss, it is necessary (though not sufficient) that the cost should fall as output increases. Then a system of multiple prices can be advantageous, when, as it is easy to verify (Fig. 8), by increasing consumption from  $M$  with a single price to  $N$  with multiple prices, with the lessening of the unit cost, the obliquely shaded surface may be bigger than the vertically shaded area. This is demonstrated in a few words, in spite of the apparent complication. Indeed, if there is a change from position  $M$  with a single price to position  $N'$ , still with a single price, the gain is the obliquely shaded area. If from position  $N'$  with a single price there is further change to position  $N$  with multiple prices, the loss (according to what we saw just now) is represented by the vertically shaded area; thus in passing from

FIGURE 8





$M$  with a single price to  $N$  with multiple prices, there is a gain represented by the first area (oblique shading) and a loss by the second (vertical shading).

Hence, when the first area is larger than the second it is possible that multiple prices may be consistent with increased welfare for the community. And as such a proceeding is more possible practically when production is socialized, this is in reality a sound argument in defence of socialized production, in certain cases, when such conditions are proved to exist.

51. Now, without rejecting the notion that in some particular case the proceeding may be applicable, the Ministry of Production, since it has under its control the determination of the  $\gamma$ 's of the supplements to income, with which it can *directly* modify distribution, does not (having regard to the practical necessity of not adding other complications to those which it must solve in the immense problem with which it is faced) think it opportune to depart from the criterion of the single price in general. At the same time it may consider some particular cases in which the multiple price system can noticeably increase the collective welfare. For example, it might treat as special cases some products for wide consumption, by extending the production of which it would make possible a considerable lessening of the cost of production.

In such cases the sale of one part under cost and of another above cost, can produce advantage for some such as could be obtained by an increase in their  $\gamma$ 's only by reducing the  $\gamma$ 's of the others by much more than the latter would lose by having to pay a price above cost.

There are cases also, in which the multiple price system, with the increase of production which it makes possible, can lead to such a lessening of cost that the new price above cost would remain below the old single price equal to the cost of production. And in such cases, of evident and great advantage to the community, nothing debars the Ministry of Production from adopting the multiple price system in place of the single price. It is an error to believe that the single price is the better system *in every case*.

52. *The Supplement to Incomes.* – The origin of all the supplements distributed to the various individuals is constituted by the price of the services of which the State has become the possessor. This sum is divided according to certain rules fixing the individual  $\gamma$ 's. It would be erroneous to conclude from this that in the collectivist régime the individuals are benefited by all that which in the old régime formed the *income* of the possessors of this capital. In fact, with the mass of all the disposable services – which, save for the different appropriation, let us suppose for the purpose of comparison, are not changed – in the old régime, the consumption of products, the consumption of services and saving for the formation of new capital were provided for. If it is desired that in the new régime existing capital should

not be destroyed and that creation of new capital should be continued at a rate no less than that which was obtained in the old régime, the community must save as formerly. Hence its consumption (of products and consumable services) would be unchanged; and thus in the new régime the community could not appropriate for consumption the *income* of the old possessors of resources, but at most only that part of this income which they consumed.

53. The distribution of that certain quantity

$$X = Q_s \lambda_s + Q_l \lambda_l + \dots,$$

the price of the services of the resources possessed by the State, can be made in many different ways. For example: *in equal parts*, making  $\gamma$  identical for all individuals; by *classes*, giving to the individual a fraction  $\gamma_1 X$ ,  $\gamma_2 X$ , etc., according to the class to which he is assigned.

It may be asked (34) if it is not possible for the Ministry of Production, in exercising its power to vary the individual  $\gamma$ 's, subject only to the condition of  $\Sigma \gamma = 1$ , to arrive at a series of  $\gamma$ 's, with the equivalents and the technical coefficients such that not only  $\Sigma \Delta \theta$  is zero but also the single  $\Delta \theta$ 's are zero. Then an absolutely indisputable maximum would be realized, because then such an economic system would be worked out, that every alteration from it in the  $\gamma$ 's, in the equivalents and in the technical coefficients would produce a decline in welfare for everyone: the ideal of economic systems. *But such a system of  $\gamma$ 's does not exist.*

In fact, the individual  $\gamma$ 's must be a function of the  $\lambda$ 's and satisfy the condition that the variation of a  $\lambda$  involves a variation of the  $\gamma$  which makes the former equal zero.

The function  $\gamma$  must therefore satisfy the conditions

$$-r_b + X \frac{\partial \gamma}{\partial \lambda_b} = 0 \dots, \quad q_m - r_m + X \frac{\partial \gamma}{\partial \lambda_m} = 0 \dots, \quad \gamma Q_s - r_s + X \frac{\partial \gamma}{\partial \lambda_s} = 0$$

(let us recollect that the individual equation is

$$r_a + \lambda_b r_b + \dots + \lambda_m r_m + \lambda_n r_n + \dots + \lambda_s r_s + \lambda_l r_l + \dots + e = \lambda_m q_m + \lambda_n q_n + \dots + \gamma X);$$

that is, it must satisfy the conditions

$$\frac{\partial \gamma}{\partial \lambda_b} = \frac{1}{X} r_b \dots, \quad \frac{\partial \gamma}{\partial \lambda_m} = \frac{1}{X} (r_m - q_m) \dots, \quad \frac{\partial \gamma}{\partial \lambda_s} = \frac{1}{X} (r_s - \gamma Q_s).$$

It is easy to see that the function  $\gamma$  which satisfies such conditions does not exist; since describing as  $\gamma_b \dots \gamma_m \dots \gamma_s$  its partial derivatives, the known conditions of integrability are not satisfied.

$$\frac{\partial \gamma_b}{\partial \lambda_m} = \frac{\partial \gamma_m}{\partial \lambda_b}; \quad \frac{\partial \gamma_b}{\partial \lambda_s} = \frac{\partial \gamma_s}{\partial \lambda_b}; \quad \frac{\partial \gamma_m}{\partial \lambda_s} = \frac{\partial \gamma_s}{\partial \lambda_m}.$$

Hence there does not exist a function of the  $\lambda$ 's, which, used for the regulating of the  $\gamma$ 's, can lead to the marvellous result that the individual  $\Delta\theta$ 's may equal zero, so that any subsequent alteration in the equivalents would cause *a decline in welfare for everyone*.

54. The effects of distribution on production would vary with the different methods by which  $X$  is distributed.

We have already noted (38) how the complete resemblance between the equations of free competition and the equations of the collectivist equilibrium, established with the idea of obtaining the maximum collective benefit, only means that there being in the group the same quantities of capital in one case as in the other, the appropriation alone being different, the economic quantities of the equilibrium will be equal to those of the other, there still being in both cases equations expressing the conditions of minimum cost and of prices equal to costs; that is precisely on account of that supplement added to the income of each individual. The distribution, which is made of that  $X$  in one way or another according to the various values which are given to the  $\gamma$ 's, influences diversely these economic quantities. The study of these diverse influences gives rise to interesting speculations, one of the most remarkable (though not unexpected) results being that there would be a sharp rise in the premium for deferred consumption – which is the parallel to *interest* on saving in the old régime – which according to most superficial collectivist doctrines would be abolished. Precisely the opposite is the case!

55. *The Equations of the Equilibrium insoluble a priori*. – For the solution of the problem it is not enough that the Ministry of Production has arrived at tracing out for itself the system of equations of the equilibrium best adapted for obtaining the collective maximum in the well-known sense (to which we need not return). It is necessary to solve the equations afterwards. And that is the problem.

Many of the writers who have criticized collectivism have hesitated to use as evidence the practical difficulties in establishing on paper the various equivalents; but it seems they have not perceived what really are the difficulties – or more frankly, the impossibility – of solving such equations *a priori*.

56. If, for a moment, we assume that the economic variability of the technical coefficients may be neglected and we take account of their technical variability only, it is not impossible to solve on paper the equations of the equilibrium. It would be a tremendous – a gigantic – work (work therefore taken from the productive services): but it is not an *impossibility*.

It is conceivable in fact that with a vast organization for this work it would be possible to collect the individual schedules for every given series of the various equivalents, including the premium for deferred consumption. Hence it is not inconceivable that with these schedules collected – always

supposing the technical coefficients known and invariable – it would be possible by a paper calculation to find a series of equivalents which would satisfy the equations expressing the physical necessities of production and the equalization of costs of production and the equivalents, which become the *prices*. There is no analytical difficulty about it: it is a problem of very simple linear equations. The difficulty arises rather from the very great number of individuals and goods of which we must take account; but it is not inconceivable that, with still more arduous work such difficulty could be overcome.

57. But it is frankly *inconceivable* that the *economic* determination of the technical coefficients can be made *a priori*, in such a way as to satisfy the condition of the minimum cost of production which is an essential condition for obtaining that maximum to which we have referred. This *economic* variability of the technical coefficients is certainly neglected by the collectivists; but that it is one of the most important sides of the question Pareto has already very clearly shown in one of his many ingenious contributions to the science.

The determination of the coefficients economically most advantageous can only be done in an *experimental* way: and not on a *small scale*, as could be done in a laboratory; but with experiments on a *very large scale*, because often the advantage of the variation has its origin precisely in a new and greater dimension of the undertaking. Experiments may be successful in the sense that they may lead to a lower cost combination of factors; or they may be unsuccessful, in which case that particular organization may not be copied and repeated and others will be preferred, which *experimentally* have given a better result.

The Ministry of Production could not do without these experiments for the determination of the *economically* most advantageous technical coefficients if it would realize the condition of the minimum cost of production which is *essential* for the attainment of the maximum collective welfare.

It is on this account that the equations of the equilibrium with the maximum collective welfare are not soluble *a priori*, on paper.

58. Some collectivist writers, bemoaning the continual destruction of firms (those with higher costs) by free competition, think that the creation of enterprises to be destroyed later can be avoided, and hope that with *organized* production it is possible to avoid the dissipation and destruction of wealth which such *experiments* involve, and which they believe to be the peculiar property of 'anarchist' production. Thereby these writers simply show that they have no clear idea of what production really is, and that they are not even disposed to probe a little deeper into the problem which will concern the Ministry which will be established for the purpose in the Collectivist State.

We repeat, that if the Ministry will not remain bound by the traditional technical coefficients, which would produce a destruction of wealth in another sense – in the sense that the greater wealth which could have been realized will not be realized – it has no other means of determining *a priori* the technical coefficients most advantageous economically, and must of necessity resort to experiments on a large scale in order to decide *afterwards* which are the most appropriate organizations, which it is advantageous to maintain in existence and to enlarge to obtain the collective maximum more easily, and which, on the other hand, it is best to discard as failures.

59. *Conclusions.* – From what we have seen and demonstrated hitherto, it is obvious how fantastic those doctrines are which imagine that production in the collectivist régime would be ordered in a manner substantially different from that of “anarchist” production.

If the Ministry of Production proposes to obtain the collective maximum – which it obviously must, whatever law of distribution may be adopted – all the economic categories of the old régime must reappear, though maybe with other names: prices, salaries, interest, rent, profit, saving, etc. Not only that; but always provided that it wishes to obtain that maximum with the services of which the individuals and the community dispose, the same two fundamental conditions which characterize free competition reappear, and the maximum is more nearly attained the more perfectly they are realized. We refer, of course, to the conditions of minimum cost of production and the equalization of price to cost of production.

60. This conclusion could have been reached, at first sight, by a “synthetic” argument; but it could not have acquired the value of a demonstrated truth, without the phenomenon being subjected to a minute quantitative analysis, as has been done in the preceding pages. The argument would be this: to hand over some capital to the State and afterwards to distribute the yield thereof among the individuals, according to a certain law, whatever it is, is like starting from a situation in the individualist régime, in which the individuals, besides having their own capital, may be possessors of certain quotas of capital of which the State has become the controller, quotas corresponding to that same law of distribution which we supposed adopted.

In such a situation what are the technical coefficients and what is the system of equivalents which allow the attainment of the maximum? Those which give the equalization of price to cost of production and the minimum cost of production!

61. That supplement to income come distributed among the individuals – whatever may be the system of distribution – does not augment, as we have seen, the consumption of products and consumable services of the group, by the total *income* which in the old régime the possessors of capital received

and which is appropriated by the State in the new régime, even when this appropriation takes place without some promise of compensation to the expropriated owners. When there is no intention of restricting saving and the creation of new capital to narrower limits than in the old régime (to this we shall return in a moment) the total consumption of products and of consumable services can be scarcely different from what it was before.

Hence, given that there is no wish to check the creation of new capital in the new régime, the distribution of consumable goods and services among the people must inevitably be restricted within the limits of what in the old régime the possessors of the capital, which is now socialized, consumed, not the whole of what they received as income. Besides this, account must be taken of the necessary remuneration of the army of officials whose services would be devoted not to production but to the laborious and colossal centralization work of the Ministry (assuming the practical possibility of such a system).

62. If it were so desired, it would be possible to augment consumption, at the expense, however, of the formation of new resources, but of *all* the new resources, even at the expense of the birth-rate. To promise increased welfare and to propose to “organize” production and to preach about free love in the new régime is simply ridiculous nonsense. If the State does not wish the collective maximum to decrease rapidly in time, the accumulation of capital must be regulated according to the birth-rate; or, conversely, the latter must be restricted within the limits set by the former.