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The Relationship between Output and Employment in British Manufacturing Industries*

I INTRODUCTION

In recent years the cyclical behaviour of the relationship between output and employment has become the subject of many interesting theoretical and empirical investigations. In the United States Dhrymes [4], Hultgren [7], Kuh [10, 11, 12], Solow [16] and Wilson and Eckstein [17] have analysed labour productivity and elaborated the implications of its cyclical pattern for income distribution, profit mark-ups, adjustment costs, etc. Similar work has been done in the United Kingdom by Godley and Shepherd [5], Neild [14] and, more recently, by Ball and St. Cyr [2]. In this paper the results of a further analysis of the British data will be presented.

We shall operate with a model which contains an underlying employment demand function and a short-term employment adjustment process. The employment demand function relates the desired level of employment to a number of exogenous variables and the adjustment process describes the adjustment of actual to desired employment. It would appear that the explicit recognition of the adjustment process is necessary, first, for the correct interpretation of actual employment figures (and, hence, of labour productivity as conventionally calculated) and, second, for the estimation of the desired level of employment. Our procedure is based on the fundamental assumption that the underlying employment demand function and the adjustment process are sufficient for the explanation of movements in employment, so that labour supply conditions and other factors are quantitatively unimportant.

In the following section a brief sketch of the model will be given. Part II is devoted to a statistical examination of the employment demand function. In part III the model will be examined for structural changes and in part IV the quantitative importance of our results will be analysed.

1. *The Employment Demand Function and the Adjustment Process*

In this section we shall give a brief outline of a model designed to describe the employment policy of a typical firm in British manufacturing industries. It will be convenient to develop the model in three sub-sections. The first deals with the determination of the firm's desired labour services, the second with its desired employment, and the third with the adjustment process.

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(*a*) *The determination of desired labour services*

We shall assume conditions of imperfect competition with administered prices. In these circumstances firms will treat their sales as exogenous in the short run because advertising and other means of encouraging demand are effective only in the long run.[1] For the sake of simplicity we shall suppose further that the firm's output bears a constant relationship to its sales.[2] Hence, output (Q) will be taken as an exogenous variable which is one of the determinants of the demand for labour services.

The second exogenous determinant of the demand for labour services is the stock of capital (K). The reason why the stock of capital is treated as exogenous is that it cannot be adjusted to changing conditions in the short run. We do not deny, however, that capital ought to be regarded as endogenous in models of long-run decisions on optimum factor proportions.

Finally, there is the state of technology (T). Following convention we shall treat it as exogenous in the short run though it may be endogenous in the long run.

The firm's demand function for labour services can now be written as:

$$E_s = f(Q, K, T) \tag{1}$$

where E_s stands for labour services. In general we would expect the partial derivatives of this function to have the following signs:

$$\frac{\partial E_s}{\partial Q} > 0, \ \frac{\partial E_s}{\partial K} < 0 \text{ and } \frac{\partial E_s}{\partial T} < 0.$$

Though we have referred to equation (1) as a demand function for labour services, it is simply an inversion of the production function which is conventionally written as $Q = g(E_s, K, T)$. Inversion and change of nomenclature do not, of course, change the production surface, but they do serve to emphasise that, in our short-run model, we regard E_s as endogenous and Q, K and T as exogenous.[3]

(*b*) *The employment demand function*

In equation (1) E_s stands for labour *services*. For the purposes of this paper it will be convenient to distinguish between two dimensions of labour services: the number of men employed (E) and the degree to which they are utilised (U). Hence, once E_s is determined by equation (1) the firm has to take a *further* decision on its desired E and U. It must be emphasised that U consists not only of the average number of hours worked per man, but also of the intensity and continuity of his efforts.

The problem of choosing the optimum E and U may be illustrated by means of the familiar iso-quant diagram. In Figure 1 E is measured along the vertical and U along the horizontal axis. The iso-quants E'_s and E''_s are the loci of the combinations of E and U which yield equal amounts of labour services. In order to discover which combination of

[1] Decisions on purchases of most labour services can be implemented and reversed within a matter of weeks or months. Hence, we treat them as essentially short-run decisions.

[2] Output is defined as *net* of materials—that is, as value added—produced by labour and capital. The analysis could be made more complex by developing it along the lines suggested by J. Johnston in his interesting study of the production decision [8].

[3] Several commentators have advised me to treat K as endogenous and operate with a reduced form which includes relative factor prices as exogenous variables. I have been reluctant to accept this advice because I feel that cost minimisation (or any other motivation) cannot be assumed as the correct behaviour over the relatively short periods studied here. More work on this problem is in progress. Dhrymes [4] assumed cost minimisation whilst Kuh [11] expressed employment as a function of output and the remaining input variables. See the interesting discussion of cost minimisation with variable effective wage rates by Ball and St. Cyr [2]. We must recognise, however, that our later statistical results will be subject to simultaneous equation bias if, in fact, K, T or Q are not exogenous.

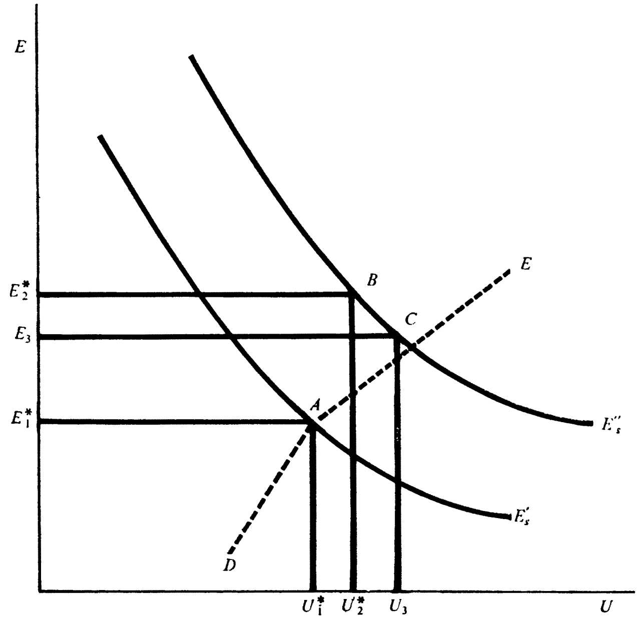

FIGURE 1

E and U the firm will choose we must make an assumption about its motivation. If short-run cost minimisation is assumed then the two optima E_1* and U_1* are determined at the point of tangency of the price line and the iso-quant E'_s. The price line has not been drawn because a little experimentation has shown that it may have many different shapes depending on how U is rewarded. There are piece rates, special bonus schemes, overtime rates, etc., all of which influence the curvature and position of the price line. A simple linear price line is most unrealistic. The problem becomes simpler, however, if we make the assumption that U is simply the average number of hours worked per man and that there are only two hourly wage rates, namely w_1 which is payable up to normal (or standard) hours (H) and w_2 which is the overtime rate. The total wage bill (W)—that is, the cost function—can then be written as $W = E(h_1w_1 + h_2w_2)$ where h_1 and h_2 are the number of hours worked for standard and overtime pay respectively. The general expression for the price line is thus given by:

(2) $$E = \frac{W}{h_1w_1 + h_2w_2}.$$

We must distinguish three different situations. First, when $h_1 < H$ and $h_2 = 0$ then equation (2) reduces to a rectangular hyperbola:

(2a) $$E = \frac{W}{h_1w_1}.$$

Secondly, when $h_1 = H$ and $h_2 = 0$ then

(2b) $$E = \frac{W}{Hw_1}.$$

Thirdly, when $h_1 = H$ and $h_2 > 0$ then equation (2) becomes

(2c) $$E = \frac{W}{Hw_1 + h_2w_2}.$$

Figure 2 is designed to illustrate the main features of the price line. The number of men employed is measured along the vertical, and average hours worked along the horizontal axis. In order to keep the geometry simple we have not drawn the relevant iso-quants.[1] We shall start by assuming an initial optimum combination of E and U at P_1, B or P_2; changes in the three exogenous variables H, w_1 and w_2 will then be assumed; the resulting shift in the position of the price line will be offset by compensating changes in W; thus the final price line will still pass through P_1, B or P_2 but its slope may have changed. If the slope has become steeper there will be a tendency to substitute men for hours and vice versa.

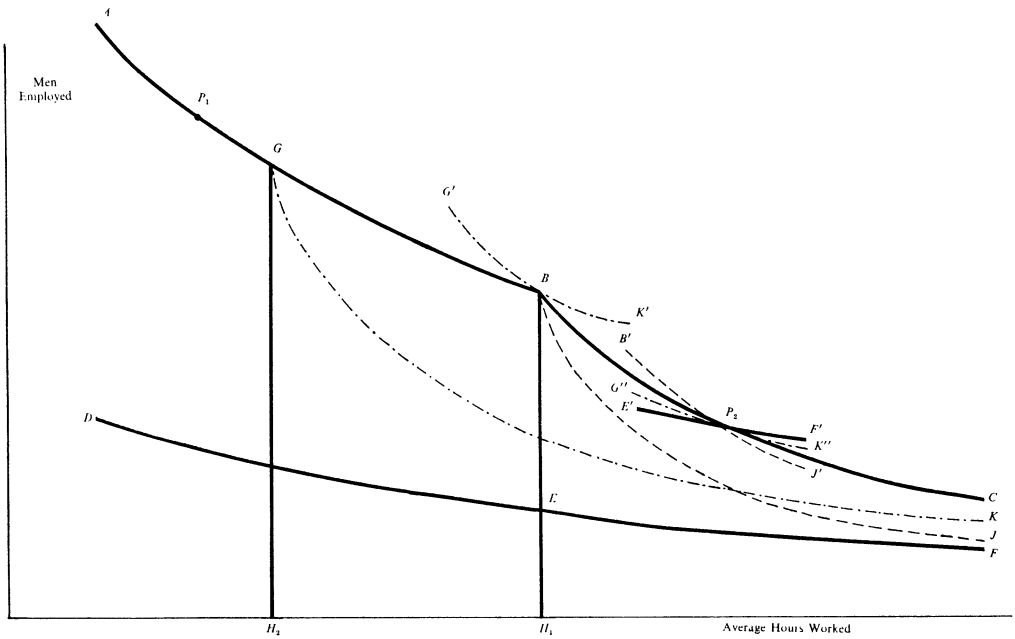

FIGURE 2

To start with, consider the price line $A - B - C$ which has a kink at B. To the left of B actual hours fall short of normal hours ($h_1 < H$) and, hence, equation (2a) holds. At B actual hours equal normal hours ($h_1 = H$, $h_2 = 0$) and, hence, equation (2b) holds. Finally, to the right of B actual hours exceed normal hours ($h_1 = H$, $h_2 > 0$) and, hence, equation (2c) holds. Let us examine the effects of changes in w_1, w_2 and H in these three cases in turn.

(i) If the initial equilibrium is at P_1, then changes in w_1, w_2 and H will leave its position unaffected because such changes do not change the slope of the price line in the neighbourhood of P_1. In the case of w_2 and H, this proposition can be proved easily: since no overtime is worked anyway, changes in the overtime rate (w_2) and normal hours (H) are quite irrelevant.[2] In the case of a rise in w_1, the price line would move, in the first instance, from $A - B$ to $D - E$ but the compensating rise in W would move the line back to $A - B$.

[1] It is important to realise that the iso-quants should *not* be rectangular hyperbolae. If they are—that is, if $E_s = Eh$—then there is no unique minimum cost solution because, to the left of the kink, the price line is also a rectangular hyperbola. A function of the type $E_S = E^{\alpha} h^{\beta}$ where $\alpha < 1$ and $\beta < 1$ would, however, yield a unique minimum cost position. See Ball and St. Cyr [2] for an alternative cost function which together with $E_s = Eh$ would yield a unique minimum cost position.

[2] This is, of course, true only for such changes in H which leave P_1 to the left of the kink.

Hence, changes in w_1 do not change the slope of the price line in the neighbourhood of P_1.[1]

(ii) If the initial equilibrium position is at B (namely at the kink) then changes in w_1 and w_2 will leave its position unaffected, but reductions in H may move its position to the left. Let us look, first, at a rise in w_1. This would move the price line from $A - B - C$ to $D - E - F$; but the compensating rise in W would move it back to $A - B - C$ where B is again the equilibrium position. Consider next a rise in w_2. This alters the price line from $A - B - C$ to $A - B - J$ and thus leaves the initial equilibrium at B unaffected. Finally, let normal hours fall from H_1 to H_2. This would change the price line from $A - B - C$ to $A - G - K$. After the compensating rise in W the relevant part of the price line is $G' - K'$ which is steeper than $A - B$ and flatter than $B - C$. Hence, not much can be said about the new equilibrium position without knowledge of the curvature of the iso-quant. If, however, the iso-quant is fairly flat then $G' - K'$ will not represent minimum costs but the equilibrium will continue to lie at the kink and thus move from B to G as normal hours fall from H_1 to H_2.

(iii) If the initial equilibrium is at P_2 then a rise in w_1 or a fall in H will move its position to the right and a rise in w_2 to the left; an equal proportionate rise in w_1 and w_2, however, will leave its position unaffected. A rise in w_1 will shift the price line from $A - B - C$ to $D - E - F$. After the compensating rise in W the relevant part of the price line is $E' - F'$ which is flatter than $B - C$. Hence, as standard pay (w_1) rises, there will be a tendency to substitute overtime hours for men.

A rise in w_2 will change the price line from $A - B - C$ to $A - B - J$. After the compensating rise in W the relevant part of the price line will be $B' - J'$ which is steeper than $B - C$. Hence, as the overtime rate (w_2) rises there will be a tendency to substitute men for overtime hours. Further, it can easily be shown that, if w_1 and w_2 rise in the same proportion, the slope of the price line remains unaffected.

A fall in normal hours from H_1 to H_2 changes the price line from $A - B - C$ to $A - G - K$. After the compensating rise in W the relevant part of the price line will be $G'' - K''$ which is flatter than $B - C$. Hence, as normal hours fall there will be a tendency to substitute overtime hours for men.[2]

[1] The point can be made in a slightly different way: at P_1 an additional man costs $h'_1 w'_1$ whilst an additional hour from already employed men costs w'_1. Hence, the price ratio $\frac{h'_1 w'_1}{w_1}$ (that is the slope of $A - B$ at P_1) remains unaffected by changes in w_1, w_2 or H.

[2] These three predictions can be derived in a slightly different way: the initial price of an additional man is $h'_1 w'_1 + h'_2 w'_2$ and the price of an additional hour from already employed men is w'_2, so that the price ratio (that is, the slope of the price line at P_2) can be expressed as:

$$\text{(a)} \qquad \frac{h'_1 w'_1 + h'_2 w'_2}{w'_2}.$$

A rise in w_1 and w_2 and a fall in H will produce the following new price ratios:

$$\text{(b)} \qquad \frac{h'_1(w'_1 + \Delta w_1) + h'_2 w'_2}{w'_2} \text{ which is larger than (a),}$$

$$\text{(c)} \qquad \frac{h'_1 w'_1 + h'_2(w'_2 + \Delta w_2)}{w'_2 + \Delta w_2} \text{ which is smaller than (a),}$$

$$\text{(d)} \qquad \frac{(h'_1 - \Delta H)w'_1 + (h'_2 + \Delta H)w'_2}{w'_2} \text{ which is larger than (a) if } w_2 > w_1.$$

Finally, let both w_1 and w_2 rise in the same proportion (p):

$$\text{(e)} \qquad \frac{h'_1 w'_1(1 + p) + h'_2 w'_2(1 + p)}{w'_2(1 + p)} \text{ which is the same as (a).}$$

Our discussion of the influence of changes in standard pay (w_1), in overtime pay (w_2) and in normal hours (H) upon the desired level of employment and average hours worked can now be summarised as follows: A rise in the ratio of overtime pay to standard pay $\left(\frac{w_2}{w_1}\right)$ either has no effect or leads to a substitution of men for hours. A fall in the level of normal hours will have no effect when actual are below normal hours; it will raise employment and lower hours when the latter are equal to normal hours; finally it will lower employment and raise hours when the latter exceed normal hours.

We are now in a position to specify our employment demand function. If U can be approximated by average hours and if firms can be assumed to be short-run cost minimisers then

(3a) $$E^* = F_1\left(E_s, H, \frac{w_2}{w_1}\right)$$

and

(3b) $$U^* = F_2\left(E_s, H, \frac{w_2}{w_1}\right)$$

describe the firm's *desired* employment (E^*) and its *desired* degree of utilisation (U^*) in terms of the exogenously given E_s, H, and $\frac{w_2}{w_1}$. We shall operate with equation (3a) which after substitution for E_s can be written as:

(3a') $$E^* = G\left(Q, K, T, H, \frac{w_2}{w_1}\right).$$

We would expect the partial derivatives to have the following signs $\frac{\partial E^*}{\partial H} \gtreqless 0, \frac{\partial E^*}{\partial \frac{w_2}{w_1}} \geqslant 0$ and $\frac{\partial E^*}{\partial E_s} > 0$. Equation (3'a) will be called underlying employment demand function. Part of it, namely $E_s = f(Q, K, T)$ is a structural equation (viz. a production function) and the other part is a reduced form.

(*c*) *The adjustment process*

Having derived our underlying labour demand function we must now deal with the adjustment process which describes the relationship between actual employment and desired employment. For this purpose we shall refer again to Figure 1. Suppose that the iso-quant rises from E_s' to E_s'' and that the new equilibrium is at B. It seems reasonable to assume that the movement from A to B will not be instantaneous. More specifically we would expect the firm to move from A to some point C and from there to B. There are, at least, two reasons why the firm should adjust its employment level slowly. First, it may become extremely expensive or even impossible to hire more than a certain maximum number of men in each period. New men may have to be attracted, trained, provided with amenities, etc., none of which can be achieved at very short notice for many men. Secondly, the firm may be uncertain about the permanency of the new level of E_s'' particularly if the latter was caused by a change in sales and, hence, output. In these circumstances the firm may be hesitant to employ more men because the possibility of their early dismissal is regarded as undesirable. For these two reasons the firm is likely to prefer initially, to raise U above U_2^* and then to move slowly to E_2^* and U_2^*.[1]

[1] Hiring and dismissal costs were introduced *explicitly* into the firm's cost function by R. M. Solow [16]. It does not appear to be easy to find a plausible cost function which produces the convenient adjustment process to be used in this paper.

We must now specify the process by which E is adjusted from E_1^* to E_2^*. A particularly convenient and simple adjustment process is the stock adjustment equation according to which:

$$\dot{E} = \alpha(E_t^* - E_{t-1}) \tag{4}$$

where $\dot{E}$ is the actual *change* in employment and α is the proportion of the difference between the desired and the actual level of employment which is eliminated in the current period.[1]

This concludes our sketch of the theoretical model which will be used in the remainder of the paper. Two important aims of our empirical work can now be stated more succinctly. First, we must examine whether there is an adjustment process of the kind postulated in equation (4) and assess its quantitative importance. Second, we shall attempt to estimate the employment demand function within the framework of the adjustment process.

2. *Preliminary Evidence on the Adjustment Process*

In his extensive study of the behaviour of British Manufacturing Industries, R. R. Neild [14] has already drawn attention to the relatively long distributed time lag of employment behind output. *For illustrative purposes* we fitted the adjustment equation (4) to the data, assuming the simplest type of employment demand function, namely:

$$E_t^* = a_0 + a_1 Q_t \tag{5}$$

so that, E_t^* is taken to depend only on output.[2] By substituting for E_t^* in equation (4) we obtain:

$$\dot{E} = \alpha a_0 + \alpha a_1\, Q_t - \alpha E_{t-}{}^{1}$$

The corresponding regression equation was:

$$\begin{array}{llll} \text{(4a)} & \dot{E} = 21.44 + .095Q_t - .308E_{t-1} & (\bar{R}^2 = .47). \\ & \qquad\;\; (3.09) \quad (.015) \qquad (.045) & (d \;\; = .63) \end{array}$$

The figures in brackets are the standard errors of the regression coefficients, and d is the Durbin-Watson statistic which gives us a measure of the first-order auto-correlation in the residuals.[3]

The regression implies that $\alpha = .308$ (α is the regression coefficient of E_{t-1}). By dividing the other two regression coefficients by α we obtain estimates of a_0 and a_1 and, hence:

$$E_t^* = 69.68 + .310Q_t. \tag{6a}$$

Our estimate of α indicates that only about 30 per cent of any difference between the desired and actual numbers of employees are eliminated in the current quarter. This implies a rather long adjustment process. Thus both Neild's evidence and equation (6) suggest strongly that there is an adjustment process which appears to be similar to the one specified in equation (4). We are, therefore, faced by a persuasive *prima facie* case for estimating the employment demand function within the framework of an adjustment process. In part II of this paper we shall see that equation (4) will continue to be a satisfactory specification of this adjustment process even when more elaborate employment demand functions are postulated.

[1] Equation (4) is, of course, formally identical to the Koyck-type distributed time lag because it can be re-written as $E_t = \alpha E_t^* + (1 - \alpha)E_{t-1}$.

[2] This linear specification of the employment demand function is unusual in empirical investigations. Among other things it implies constant marginal products over the range covered by the observations. Appendix A contains the results of our regressions when the models are specified in log-linear form.

[3] The data were seasonally adjusted quarterly index numbers for employment and output in British manufacturing industries (1958 = 100) for the period 1950, I to 1963, IV (52 observations). See Appendix B.

3. *A Statistical Reason for the Observed Lags*

In section I,1 we sketched a theory which predicted that employment should react only slowly to changes in output. Another possible reason for this kind of lag is a purely statistical one: it would appear that the index of manufacturing production contains some delivery series which may not have been adequately adjusted for stock changes. A rise in deliveries may lead to a reduction in stocks which *with a lag* raises output which, in turn, *without a lag* raises employment. Hence, our observed lag between the index of production and employment may, in fact, be a lag between deliveries and output. We shall present some evidence in a later section which casts some slight doubt upon this explanation of the lag. In general, the remainder of this paper is based on the assumption that this statistical reason for the lag is quantitatively unimportant.[1]

4. *Men and Man-Hours*

In section I,1 we suggested that labour services and, hence, output may rise without an immediate rise in the level of employment because the existing labour force may be induced to work harder and longer hours. It may be argued that we ought to measure labour demand in terms of total man-hours rather than number of employees because a large part of the adjustment process would thereby disappear.

One of the reasons why we chose employment rather than man-hours was that we wanted to discover the relationship between output and unemployment. Since unemployment is measured in men we decided to measure employment in men as well. The choice between men and man-hours, however, depends primarily on the aims of the investigator. Hence, we have re-run the most important regressions in terms of man-hours and present them in Appendix A.

II ESTIMATION OF THE EMPLOYMENT DEMAND FUNCTION

Equations (4a) and (6) imply an employment demand function which is of the simplest possible type, namely $E^* = a_0 + a_1Q_t$. This part of the paper is devoted to estimating more elaborate employment demand functions of the type given in equation (3′a).[2] We shall proceed step by step and introduce successively:

(i) a time trend which may be taken to represent technological change,
(ii) an estimate of the capital stock,
(iii) normal hours, and
(iv) expectations of future levels of output.

A final section will deal with the special problem of increasing and decreasing marginal returns to *ceteris paribus* changes in the level of output.

1. *Time Trends*

The use of time trends in the employment demand function may be justifiable if we wish to take into account certain variables for which we have no data and which are expected to vary smoothly over time. Frequently technological and organisational progress is treated as such a variable. Let us postulate that T is a quadratic function of time (t).

[1] From many a practical point of view it may not matter whether we measure the lag between deliveries and output or that between output and employment. Trouble would, however, arise if the lags changed significantly and we wanted to discover the reasons for such changes.

[2] Neild [14], who also dealt with British manufacturing industries, did not specify his employment demand functions explicitly. He related log E to log Q with various lag structures and then repeated the operations with first differences of log E and log Q. Thus his first implied employment demand function relates employment only to output (as in our equation (6)). His second implied function relates employment to output and a logarithmic time trend (which shows up in the constant term). Neild is not particularly satisfied with his regressions, since he concludes: " These are not very good results ", (p. 33). We shall attempt to improve on these results by postulating more elaborate employment demand functions.

The employment demand function can then be written as:

$$E_t^* = a_0 + a_1 Q_t + a_2 t + a_3 t^2 \tag{7}$$

where a_2 and a_3 are expected to be negative. The t^2-term is used in order to allow for the possibility that technological progress accelerates over time.

We fitted equation (7) within the framework of the dynamic adjustment process.[1] It turned out as follows:[2]

$$\underset{}{\dot{E}} = \underset{(2.61)}{14.22} + \underset{(.014)}{.172 Q_t} - \underset{(.015)}{.028 t} - \underset{(.0002)}{.0007 t^2} - \underset{(.033)}{.297 E_{t-1}} \quad \begin{matrix} (\bar{R}^2 = .76) \\ (d = 1.37) \end{matrix} \tag{8}$$

Hence

$$E_t^* = 47.90 + .580 Q_t - .094 t - .0024 t^2. \tag{8a}$$

Compared with equation (6) the introduction of the quadratic time trend improves the overall fit quite considerably (the $\bar{R}^2$ rises from .47 to .76). The regression coefficients of t and t^2 have the expected sign and they are fairly significant. Further, the Durbin-Watson statistic rises from .63 to 1.37 which shows that positive auto-correlation in the residuals has declined.[3] Finally, the adjustment coefficient α is virtually unchanged by the introduction of t and t^2.

The result of this regression equation tends to support our arguments that some variable or set of variables, which moves smoothly over time, has an important influence on the desired level of employment (E_t^*). This variable may well be technological progress. It is interesting to note that (since the t^2-term is very significant) the rate of technological progress (or such other variables as t might represent) appears to be accelerating.

2. *The Stock of Capital*

Since we do not have a quarterly index of the capital stock in British Manufacturing we must construct our own expression for it. The present section will, therefore, be divided into two parts. First, we shall derive a formula for the capital stock which will have the form of $K_t = b_0 + b_1 t + b_2 t^2 + R_t$ where K_t is the estimated capital stock, t stands for time, R_t for the residuals of the sum of gross investment from its quadratic time trend, and the coefficients b_0, b_1 and b_2 are unknown constants. The crucial assumption which allows us to derive this expression for capital is that, though gross investment fluctuates cyclically, retirements of equipment are likely to be smooth because gross investment consists of pieces of equipment with widely varying economic lives. Secondly, we shall estimate the effect of the capital stock (as measured by our expression) upon the desired level of employment.

[1] In this and the following regressions, $t = 1$ for the first quarter 1950; $t = 2$ for the second quarter 1950 and so on until $t = 52$ for the fourth quarter 1962.

[2] R. M. Solow fitted the same relationship to U.S. Data. His regression turned out as follows:

$$\dot{E} = \underset{(3.07)}{21.22} + \underset{(.022)}{.377 Q_t} - \underset{(.025)}{.171 t} - \underset{(.0005)}{.0028 t^2} - \underset{(.036)}{.520 E_{t-1}} \quad (R^2 = .88)$$

I am much indebted to him for making his results available to me. See also footnote 1, p. 200.

[3] The absolute level of α may, however, still be biased upward because E_{t-1} is one of the independent variables.

(a) Derivation of an expression for capital

Like some other writers we define the capital stock as the sum of gross investment over the average life of the capital goods.[1] Thus if n is the average life of the equipment then

$$K_t = \sum_{j=t-n}^{t} I_j \tag{9}$$

where I_j is gross investment measured at constant prices and K_t the capital stock. This definition of the capital stock assumes that there is no noticeable *physical* deterioration of the capital goods and that the reason for scrapping equipment is not its physical collapse but the end of its economic life (namely obsolescence).

Unfortunately we do not know the value of n, and, hence, we cannot calculate K_t directly. For this reason a more indirect method must be adopted. Consider the sum of gross investment from some arbitrary period of time k; this can be written as:

$$K_t' = \sum_{j=k}^{t} I_j. \tag{10}$$

Now suppose that at the end of period 0 we define the difference between K and K' as ε:

$$K_0 - K_0' = \varepsilon. \tag{11}$$

During period 1 a certain amount of gross investment (I_1) is undertaken and added to *both* K_0 and K_0'. At the same time, however, some amount of capital (I_1^r) is *retired* and this amount is deducted from K_0 but *not* from K_0'. Thus at the end of period 1 we have:

$$K_1 = K_0 + I_1 - I_1^r$$

and

$$K_1' = K_0' + I_1.$$

Hence, the difference between the capital stock and the sum of gross investment at the end of period 1 is:

$$K_1 - K_1' = (K_0 - K_0') - I_1^r = \varepsilon - I_1^r. \tag{11a}$$

Similarly at the end of period 2 we have

$$K_2 - K_2' = \varepsilon - I_1^r - I_2^r. \tag{11b}$$

In general:

$$K_t - K_t' = \varepsilon - \sum_{j=1}^{t} I_j^r. \tag{11c}$$

Equation (11c) tells us that the difference between the capital stock (K_t) and the sum of gross investment (K_t') diminishes at the rate at which capital is retired.

[1] See the interesting discussion by Griliches [6].

We now make the crucial assumption that *retirements of capital grow at a constant arithmetic rate over time.* In symbols this assumption can be written as:

(12) $$I^r_j = I^r_0 + dj$$

where d is the arithmetic rate of growth of retirements. Substituting this expression for I^r_j in equation (11c) and re-arranging, we obtain: [1]

(13) $$K_t = K'_t + \varepsilon - \left(I^r_0 + \frac{d}{2}\right) t - \frac{d}{2} t^2.$$

Equation (13) describes the basic relationship between capital and the sum of gross investment. However, if expression (13) were used in our regressions then the relatively large coefficients of t and t^2 in (13) would overshadow those coefficients of t and t^2 which describe technological progress. *Purely for purposes of exposition* let us, therefore, reformulate equation (13). Suppose that gross investment is made up of a constant trend and a cylical component:

(14) $$I_j = I_0 + d'j + \mu_j$$

where I_0 is some initial level of gross investment, d' is its constant arithmetic growth rate and μ_j is the cyclical component.[2] The sum of I_j from I_0 to I_t is, therefore, e qual to:[3]

(15) $$K'_t = \sum_{j=1}^{t} I_j = \left(I_0 + \frac{d'}{2}\right) t + \frac{d'}{2} t^2 + \sum_{j=1}^{t} \mu_j.$$

If we now substitute this expression for K_t in equation (13) we obtain:

(13a) $$K_t = b_0 + b_1 t + b_2 t^2 + R_t$$

where
$$b_0 = \varepsilon,$$
$$b_1 = (I_0 - I^r_0) + \frac{d' - d}{2},$$
$$b_2 = \frac{d' - d}{2},$$
$$R_t = \sum_{j=1}^{t} \mu_j.$$

[1] $$K_t = K'_t + \varepsilon - \sum_{j=1}^{t} (I^r_0 + dj).$$

That is $$K_t = K'_t + \varepsilon - I^r_0 t - \sum_{j=1}^{t} dj.$$

Since $$\sum_{j=1}^{t} dj = d(1 + 2 + 3 + \ldots + t) = \frac{t(t+1)}{2} d$$

we obtain $$K_t = K'_t + \varepsilon - I^r_0 t - \frac{t(t+1)}{2} d$$

which is the same as equation (13).

[2] The assumption of a constant time trend in real gross investment is not unrealistic as inspection of our gross investment series in Appendix B shows.

[3] $$K'_t = \sum_{j=1}^{t} I_j = I_0 t + (1 + 2 + 3 + \ldots + t)d' + \sum_{=1}^{t} \mu_j.$$

Since $(1 + 2 + 3 + 4 + \ldots + t) = \frac{t(t+1)}{2}$ we obtain (15).

Compared with equation (13) the present formulation has the advantage that the coefficients b_1 and b_2 are relatively small since they are derived from the differences between I_0 and $I_0{}^r$ and between d' and d. In a growing economy we would expect the arithmetic trend of investment (d') to exceed the arithmetic trend of retirements (d), so that b_2 is likely to be positive. On the other hand, in period 0 gross investment (I_0) may be smaller than retirements ($I_0{}^r$) if period 0 happens to lie in a cyclical depression. Consequently, the coefficient b_1 may be negative.

Before equation (13a) can be used, an estimate of the cyclical component of the capital stock (namely $R_t = \sum_{j=1}^{t} \mu_j$) has to be obtained. For this purpose we fitted a quadratic time trend to the observed sum of gross investment (K_t') and then calculated the residuals (R_t). Thus in essence we estimated the coefficients of t and t^2 in equation (15) and then obtained the R_t as residuals.[1]

The basic assumptions which permitted us to derive equations (13) and (13a) were, first, that gross investment and retirements had constant arithmetic time trends (d' and d) and, second, that the retirement of capital was a fairly smooth process which approximated its trend. We must now give some reasons why these assumptions seem to be plausible and, then show what would happen if the second assumption were inappropriate.

The first assumption can be justified fairly easily on empirical grounds. We have inspected various gross investment series for British manufacturing industries and have found that they have an approximately linear time trend. For this reason our assumption that both gross investment and retirements have a constant linear time trend may not be too far off the truth.[2]

The plausibility of our second assumption derives from the fact that gross investment consists of capital goods with widely varying lives. Let us define the range of lives of capital goods as $n_2 - n_1 = r$ and let us assume that the life distribution of gross investment is rectangular, so that $\frac{I}{r}$ has a life of n_1, a further $\frac{I}{r}$ one of $n_1 + 1$ and so on. If I displays a cycle then it gives rise to r retirement cycles, each one with an amplitude of $\frac{I}{r}$ and the same frequency as the investment cycle. But the replacement cycles do not coincide; they are displaced over time and, hence, when summed they must yield a smoother retirement cycle than the original investment cycle. The larger the range of lives (r) the smoother will be the retirement process. The actual lives of capital goods seem to range between sixteen and fifty-five years.[3] Some simple arithmetical calculations have shown that this sort of range would give rise to a very smooth retirement process. Hence, our assumption that retirements have no cycle may not generate an unduly large error.

[1] We must emphasise again that the difference between using equations (13) and (13a) is one of expositional convenience and not of substance. Equation (13) can be re-written as:

(13b) $$K_t = c_0 + c_1t + c_2t^2 + K'_t.$$

Contrast this expression with (13a): $K_t = b_0 + b_1t + b_2t^2 + R_t$ where R_t is derived as a residual $R_t = K'_t - f_0 - f_1t - f_2t^2$. After substitution (13a) can be written as (13c) $K_t = (b_0 - f_0) + (b_1 - f_1)t + (b_2 - f_2)t^2 + K'_t$ which is equivalent to (13b).

[2] In the very long run, a linear time trend in gross investment implies the *same* linear time trend for retirements (that is, $d' = d$). If gross investment rises by a trend value of, say, 5 units this year, then trend retirements must rise also by 5 units after the average life of the equipment has elapsed (say, after 30 years). Our analysis does not exclude the possibility that $d' = d$. But linear time trends and $d' \neq d$ may be a satisfactory approximation over periods of, say, 10 years.

[3] See Dean's estimates of the lives of equipment in Table 1 of the Appendix of [3].

Hitherto we have assumed that the retirement process is unaffected by current economic conditions. Since retirements are not caused by the physical collapse of the equipment but by obsolescence, we must now examine how current economic conditions might influence the rate of retirements. One highly plausible assumption is that retirements will be retarded when capital is scarce and accelerated when capital is abundant. Hence, in booms the net additions to capital would be gross investment plus retarded retirements minus trend retirements and in depressions the net additions to capital would be gross investment minus accelerated retirements minus trend retirements. Our estimate of the capital stock ignores retarded and accelerated retirements and, hence, it may understate the capital stock in investment booms and overstate it in the investment depressions. But our estimate of the capital stock will still peak and trough at the same times as the true capital stock. *Hence, our procedure may underestimate the amplitude of fluctuations of the capital stock but its other cyclical properties (for example, frequency and turning points) are unlikely to be affected.*

To sum up: In this subsection we have derived a formula for the capital stock in terms of the deviations of the sum of gross investment from its quadratic time trend (R_t) and a quadratic time trend (see equation (13a)). For this purpose we made the basic assumption that retirements of equipment grow smoothly along their arithmetic time trend. This is not an implausible assumption because, although gross investment may have a cycle, it consists of capital goods which will be retired at varying future dates.

If retirements are not smooth, however, then they are likely to vary *inversely* with gross investment. In that case our expression will underestimate the amplitude, but reflect correctly the other cyclical properties of the actual capital stock series.

(*b*) *Capital as an explanatory variable in the employment demand function*

We must now proceed to estimate the influence of the capital stock upon desired employment. For our calculations two series for the sum of gross investment K_t' were used: first, the sum of gross fixed capital formation in manufacturing industries at 1958 prices and second the sum of factory completions in manufacturing measured in millions of square feet.[1] The results of the second series were better than those of the first. There are, at least, two reasons why factory completions should be a more satisfactory index of investment than expenditure. In the first place, there are no quarterly expenditure series for pre-1955 years and our interpolations may be unsatisfactory. Second, investment expenditure does not necessarily coincide with the initiation of the equipment as capital. Payments are made in advance as well as with varying lags after the instalment of the capital. Factory completions, on the other hand, are said to be recorded at the time when all plant and machinery is ready to go into operation. Consequently we would expect more variance in the series of factory completions than in investment expenditure. In fact, for the period 1955-62 the coefficients of variations of investment expenditure and factory completions are .13 and .16 respectively. After removal of a simple time trend the coefficients become .009 and .16 respectively. Clearly, a lot of arguments could be produced both for and against the suitability of either series. In what follows R_t refers to factory completions mainly because this series happens to give slightly more significant results.[2]

According to equation (13a) the stock of capital can be expressed in terms of R_t and a quadratic time trend. Since we have already used a quadratic time trend as a proxy variable for technological progress all we need do is to add R_t to equation (8):

$$(16) \quad \underset{}{\dot{E}} = \underset{(2.62)}{16.45} + \underset{(.014)}{.164Q_t} - \underset{(.0145)}{.0158t} - \underset{(.00019)}{.00074t^2} - \underset{(.0064)}{.0165R_t} - \underset{(.032)}{.314E_{t-1}} \qquad \begin{array}{l}(\bar{R}^2 = .79) \\ (d = 1.48)\end{array}$$

[1] Both series were seasonally adjusted. Source: *Economic Trends*.

[2] However, even when R_t is computed from investment expenditure, its regression coefficient is significant at the 5 per cent level.

The implied employment demand function is:

(16a) $$E_t^* = 52.36 + .522Q_t - .0503t - .00236t^2 - .0524R_t.$$

Compared with equation (8) the introduction of the expression for capital has improved the fit.[1] The $\bar{R}^2$ has risen slightly (from .76 to .79) and the regression coefficient of R_t is quite significant (the ratio of the coefficient to its standard error being about 2.6). Moreover, a comparison of the two d statistics shows that the introduction of R_t lowers the positive auto-correlation in the residuals. On the other hand, the negative influence of t is made less significant by the introduction of R_t.[2]

In view of the many good reasons for supposing that the influence of capital should not be noticeable in quarterly data and that our expression for the capital stock is faulty, it is perhaps rather surprising to find that capital has the expected sign and that its influence is fairly significant.[3] Moreover, there do not seem to be any very obvious reasons—other than the traditional theory presented here—why E_t^* and R_t should be negatively correlated.

3. *The Influence of Normal Hours*

In section I,1 we suggested that the level of normal hours (H) and the ratio of overtime to standard pay $\left(\frac{w_2}{w_1}\right)$ are likely to influence E^*. Unfortunately we do not have a series for $\frac{w_2}{w_1}$ and, hence, its influence cannot be ascertained. It is often assumed, however, that this ratio does not fluctuate much cyclically; in that case its absence from the regression need not cause much concern.[4]

However we do have a series for normal hours.[5] The regression with normal hours but without capital turned out as follows:

(17) $$\underset{}{\dot{E}} = \underset{(10.00)}{55.59} + \underset{(.013)}{.194Q_t} + \underset{(.0211)}{.0444t} - \underset{(.00043)}{.00241t^2} - \underset{(.078)}{.332H_t} - \underset{(.039)}{.408E_{t-1}} \qquad (\bar{R}^2 = .83) \quad (d = 1.84)$$

(17a) $$E_t^* = 136.39 + .477Q_t + .109t - .0059t^2 - .815H_t.$$

Compared with equation (8) the introduction of H_t improves our fit very noticeably. The $\bar{R}^2$ rises from .76 to .83 and the ratio of the regression coefficient of H_t to its standard error is more than 4. Moreover the auto-correlation of the residuals is much reduced. Two other differences between equation (17) and earlier results may be worth noting. The reaction coefficient α is now slightly higher (.4 rather than .3). Second, the coefficient of t is now positive and fairly significant.

[1] R. M. Solow's regression for U.S. data turned out as follows:

$$\dot{E} = \underset{(3.09)}{22.11} + \underset{(.024)}{.329Q_t} - \underset{(.026)}{.183t} - \underset{(.0005)}{.0029t^2} + \underset{(.0008)}{.0012R_t} - \underset{(.038)}{.540E_{t-1}}. \quad (R^2 = .88).$$

Solow's series for R_t was constructed on roughly the same principles as the one used in this paper.

[2] Compared with equation (8) the absolute value of the coefficient of t has fallen whilst that of t^2 has remained virtually constant. On certain assumptions this means that d' can be taken to be approximately equal to d and that I_0 falls short of I_0^r. See the discussion on p. 198.

[3] One of the commentators suggested that the fit could be improved by the introduction of the actual number of average hours worked as a proxy for the degree of capital utilisation. This was tried, but proved to be unsuccessful.

[4] For instance Neild [14], p. 64 assumes $\frac{w_2}{w_1}$ to be constant.

[5] This index covers all workers and not only manufacturing workers. For later years series are available for both " all workers " and manufacturing and they move very closely together. For pre-1958 annual figures were used.

We must now turn to an interpretation of the highly significant negative coefficient of H_t. In section I,1 it was argued that, in the case of a short-run cost minimiser, a fall in normal hours will lead to a rise in E^* only if the equilibrium point lies at the kink of the price line and this would mean that desired hours (H^*) are invariably equal to normal hours (H).[1] At first sight this implication of the theory may be regarded as inconsistent with the observation that, throughout the sample period, actual hours have exceeded normal hours; so that entrepreneurs can hardly be said to desire the latter. Our analysis of section I,1 may nevertheless be appropriate for two reasons. First, the series of actual hours worked tends to be biased upwards because certain types of short-term operations are not recorded. Secondly, the firms' desired hours (H^*) may not be equal to but bear a constant relation to normal hours (H). In British industry some amount of overtime seems to be regarded as part of the normal conditions of employment which employees cannot avoid. In this case the entrepreneurs' desired hours (H^*) may be a positive function of normal hours. An attempt has been made to analyse the empirical relationship between H^* and H by treating actual hours as the sum of desired and undesired hours and then relating desired to normal hours. Preliminary estimates indicate that the elasticity of H^* with respect to H is approximately .5.

For both these reasons our evidence may be compatible with the analysis presented in section I,1. It may, of course, be also compatible with some other theory.[2]

Having established the plausibility of a negative coefficient of H_t we must now deal briefly with its absolute size. According to equation (17a) a 1 point fall in the index of normal hours (H) leads to a rise of .8 points in the desired employment index (E^*). Since output Q is held constant, this means that output per head falls by approximately .8 per cent.[3] Further, if the elasticity of H^* with respect to H is .5, then the output per desired man-hour falls by .3 per cent. These reductions in output per head and output per desired man-hour may be regarded as unexpectedly large and they should be taken as *prima facie* evidence against the widely held view that reductions in hours worked do not lead to reductions in output per head or per man-hour.[4] [5]

There are, however, two statistical reasons why the coefficient of H may conceivably be biased. First, the time series of normal hours varied very smoothly during the sample period. It remained virtually constant until 1958. Thereafter it declined, first, slightly, and then (during 1960/61) quite substantially. It is hoped to re-run the regressions in the near future because recently the time path of normal hours has not been quite so smooth. Second, at the time when normal hours fell substantially (that is, during 1960/61) the composition of the labour force changed owing to the increased number of school-leavers. The rise in the proportion of young untrained workers may have reduced the average quality of labour and, hence, the average output per head. Whether and to what extent this change in the quality of labour has affected our estimate of the coefficient of H_t can be ascertained only by further empirical research. Until such research is undertaken we must confine ourselves to indicating the possibilities of bias.

[1] The same assumption has been made by Kuh. See [11] section II, B.

[2] In section I, 1 it was assumed that short-run cost minimisation was the only determinant of E^* and H^*. In a more complete model, workers' preferences, trade union activities, etc., must clearly play a part.

[3] Since E, Q and H are measured in index numbers (1958 = 100) the coefficient of H of .8 can be taken to approximate an elasticity.

[4] In a recent study, [1], pp. 115-117 and 128 Åberg resports that, in Swedish manufacturing, the partial elasticity of output with respect to hours worked is in the range of .69 to .76. This book seems to be the most comprehensive available study of the effects of reductions in working hours. It contains a summary in English and its contents are discussed in English by Österberg [15].

[5] Godley and Shepherd, [4], p. 35, assumed that the elasticity of output per head with respect to normal hours was .2. In view of the evidence presented in this paper and that found by Åberg [1] this assumed figure seems to be too low.

We must now combine equations (16) and (17) and admit of a joint influence of capital and normal hours upon $\dot{E}$ and, hence, E_t^*. When both H_t and R_t are introduced as independent variables we obtain:

$$\begin{aligned}(18)\qquad \dot{E} &= \underset{(13.63)}{58.79} + \underset{(.017)}{.198Q_t} + \underset{(.0243)}{.0485t} - \underset{(.00060)}{.00255t^2} + \underset{(.0085)}{.0030R_t} \\ &\quad - \underset{(.114)}{.361H_t} - \underset{(.043)}{.414E_{t-1}} \qquad (\bar{R}^2 = .82) \quad (d = 1.87)\end{aligned}$$

$$(18a)\qquad E_t^* = 141.97 + .4775Q_t + .117t - .00616t^2 + .0072R_t - .873H_t.$$

In most respects the results of (18) are similar to those of (16) and (17). But one difference stands out: the joint introduction of H_t and R_t reverses the sign of the coefficient of R_t and reduces its statistical significance substantially.

The statistical reason for this phenomenon is a very strong intercorrelation between R_t and H_t when t and t^2 are taken into account. Thus:

$$(19)\qquad H_t = \underset{(.182)}{99.120} + \underset{(.0167)}{.1494t} - \underset{(.0003)}{.0044t^2} + \underset{(.0102)}{.0580R_t} \qquad (\bar{R}^2 = .93) \quad (d = .28)$$

where the right-hand side of the equation might be taken to represent our expression for the capital stock.[1]

It is necessary for us to decide whether this positive relationship between H_t and R_t is due to economic factors or whether it is simply accidental. There is not much accepted theory on this subject [2] and, hence, the following possible explanation of our observation is rather *ad hoc*.

Our regressions suggest that, with a constant level of output, as H falls the capital stock falls also and, hence E^* rises for two reasons: (i) because desired hours (H^*) are falling and (ii) because labour is substituted for capital, that is, the capital-labour ratio is falling. This may be brought about by the following process: the fall in normal hours may induce entrepreneurs to reduce overtime working and to introduce multiple shift working instead. The same amount of output might thus be produced by half the capital stock with twice the number of employees working in two shifts. Thus, if the fall in H induces employers to reduce their capital stock (by not replacing it) and to work the remaining capital stock more intensively then our observed intercorrelation between H and R could occur.

However, this possible explanation seems rather implausible when we consider that our results were generated by quarterly data. For this reason we shall recognise the possibility that the inter-correlation between H and R is an unfortunate accident, and, hence, we shall report all the statistical tests in the framework of both equations (16) and (17).[3]

4. *The Influence of Changes in Output*

It is a widely held view that firms build up their labour requirements in anticipation of high levels of activity. This proposition might be tested by letting E^* depend upon recent *changes* in output. Let us postulate that:

$$(20)\qquad Q_{t+1}^e = Q_t + \beta\dot{Q}_t'$$

[1] When R_t is regressed on t, t^2 and H_t the overall relationship is somewhat weaker, but the coefficient of H_t is still highly significant:

$$R_t = \underset{(120.99)}{-689.54} - \underset{(.258)}{1.039t} + \underset{(.0063)}{.0307t^2} + \underset{(1.220)}{6.957H_t} \qquad (\bar{R}^2 = .37) \quad (d = .26)$$

[2] See, however, the interesting study by R. Marris [13].

[3] A comparison of (18) with (17) shows that all coefficients remain virtually unaffected by the introduction of R. The same is true of all the other tests to be reported below. Hence, all further results are not reported for equation (18).

where Q^e_{t+1} is the output expected in period $(t+1)$ and $\dot{Q}'_t$ is the recent *change* in output. According to equation (20) intrepreneurs expect future output to be equal to to-day's output plus some proportion (β) of the recent change in output. β may exceed unity or be negative.

If Q^e_{t+1} is an important determinant of E^*_t and if Q^e_{t+1} is determined according to equation (20), then we would expect $\dot{Q}'_t$ to have a significant influence in equations (16) and (17). As an approximation to recent changes in output we used not only the first differences of output ($\dot{Q}_t$) but also a moving average ($\bar{\dot{Q}}_t$) of the past four quarters of these first differences. The introduction of either of these two variables into equations (16) and (17) did not improve the overall fit significantly. The regression coefficients of $\dot{Q}_t$ and $\bar{\dot{Q}}_t$ and their standard errors are given in Table 1.

TABLE 1

	Regression Coefficient of $\dot{Q}_t$	*Regression Coefficient of* $\bar{\dot{Q}}_t$
For equation (16b) [1]	−.00929 (.03447)	.0410 (.0882)
For equation (17b) [1]	−.0397 (.0319)	.0207 (.0782)

These regression coefficients are very insignificant, and, hence, we must conclude that our evidence does not support the proposition that firms raise their desired level of employment E^* in anticipation of higher levels of output. This conclusion holds only if $\dot{Q}$ and $\bar{\dot{Q}}$ are approriate expectational variables.[2]

The result that $\dot{Q}$ and $\bar{\dot{Q}}$ do not have a significant influence upon our relationships is relevant for the evaluation of another argument. In section I,3 we asserted that, if Q measured deliveries rather than production (P) then the observed lag between Q and E may be due to the lag between Q and P and *not* to that between P and E. Production can be defined as the sum of deliveries and stock changes ($\dot{S}$) so that if Q measures deliveries

(21) $$P = Q + \dot{S}.$$

Let us postulate a constant relationship between stocks and deliveries $k = \frac{\dot{S}}{\dot{Q}}$, to that

(21a) $$P = Q + k\dot{Q}.$$

Since employment is related to production we would expect E^* to correlate with Q *and* $\dot{Q}$. Our finding, that it correlates with Q but *not* with $\dot{Q}$ can be interpreted in two ways: either Q does *not* measure deliveries and is approximately equal to production, or the accelerator-type inventory theory is not valid. Hence, our evidence casts some slight doubt upon the hypothesis that Q measures deliveries rather than production.

[1] Equations (16b) and (17b) are identical with (16) and (17) except that they contain either $\dot{Q}_t$ or $\bar{\dot{Q}}_t$ as additional variables.

[2] Kuh was able to sub-divide his employment series into production workers and overhead workers. For production workers the coefficients of changes in output are positive and significant, whilst for overhead workers the coefficients are negative but insignificant. Hence our findings which are based on all workers need not necessarily be incompatible with Kuh's plausible results. See [11], Table 9.

There are undoubtedly other theoretical models which predict that changes in output ought to affect the relationship between employment and output. The evidence presented here, however, does not support such theories.

5. *The Influence of Output*

One of the features of equations (16a) and (17a) to which we have not yet drawn attention, concerns the size of the coefficients of Q_t. At .52 and .48 they may be regarded as unexpectedly low. Thus for every ten points that the output index rises the index of E_t^* rises by only five points and, hence, output per head rises by approximately 5 per cent. In this section we shall attempt to analyse in some detail the effects of *ceteris paribus* changes in output upon desired employment.

Conventional economic theory predicts that the relationship between E^* and Q will have the same shape as curve $A - B$ in Figure 3. At low levels of Q, $\frac{\partial^2 E^*}{\partial Q^2}$ is negative and at high levels of Q, $\frac{\partial^2 E^*}{\partial Q^2}$ is positive. The ratio $\frac{E^*}{Q}$ will fall up to Q_0 and rise thereafter.

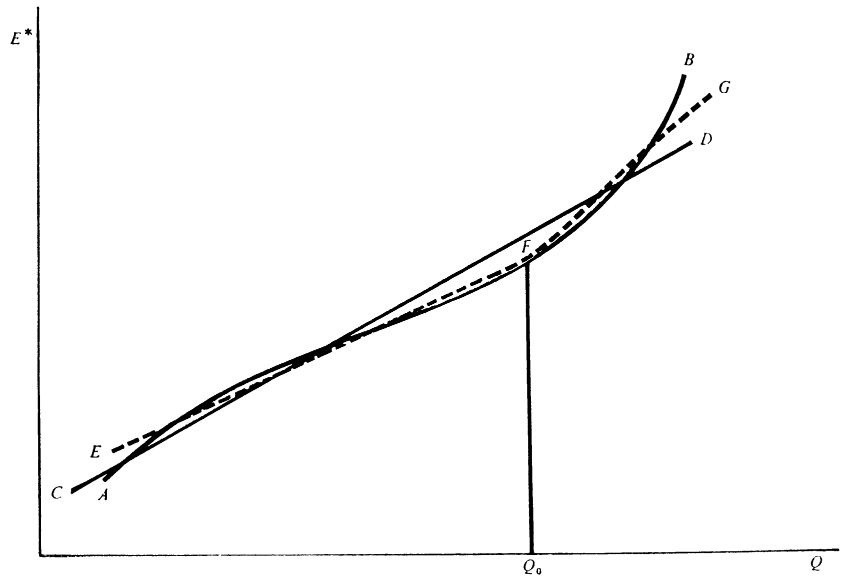

FIGURE 3

if we fitted a *linear* regression to $A - B$, however, we would obtain a line $C - D$ which would show a declining ratio $\frac{E^*}{Q}$ at all levels of Q. In other words, our basic assumption of linearity may have given us an estimate of $\frac{\partial E^*}{\partial Q}$ which is too high at low levels of Q and too low at high levels of Q.

One way of overcoming the disadvantages of the assumed linearity is by the introduction of a kink. Suppose we would allow $C - D$ to have a discontinuity at Q_0. It would then become $E - F - G$ where $E - F$ should be flatter than $F - G$. The kink should be introduced at that level of Q where the angle formed by $E - F - G$ is at a minimum (that is, where the kink is sharpest). This level of Q is quite likely to coincide with Q_0 where $\frac{E^*}{Q}$ is at a minimum. For the sake of simplicity we shall argue as though these two values of Q do coincide.

A kink could easily be introduced into the relationship between E^* and Q if we had an independent measure of Q_0. None exists, however, and, hence we have attempted to overcome this difficulty by making various assumptions. To start with Q_0 will not be constant over time, but will rise because of capital accumulation and technological progress. We assumed that Q_0 would bear a simple relationship to the trend value of output:

(22) $$Q_{0t} = Q_{et} + L$$

where Q_{0t} is the trend value of output and L is some constant. A little experimentation showed that a simple arithmetic time trend seemed to fit output in manufacturing at least as well as more elaborate formulation.[1] Hence, the trend value of output was obtained by the following equation:

(23) $$Q_{et} = 78.15 + .764t$$

and, therefore

(22a) $$Q_{0t} = 78.15 + .764t + L.$$

Thus Q_0 is assumed to grow at the same rate as Q_e, but their constant terms differ by L.

In order to be able to see whether positive deviations from Q_0 bear a steeper relationship to E^* than negative ones (see Figure 3) we computed these deviations, and then classified them according to their signs. The deviations (dQ) were derived by the following formula:

(24) $$dQ_t = Q_t - Q_{0t} = Q_t - 78.15 - .764t - L.$$

Nine different values of L were assumed namely $-4, -3, \ldots, +3, +4$ (index number points) and, hence, nine sets of alternative estimates of dQ were obtained.

The deviations were now classified according to their sign. Thus two time series were constructed from each set of deviations, one for positive ones (dQ^+) and one for negative ones (dQ^-). For each particular quarter there is only either a dQ_t^+ or a dQ_t^- and, hence, the other must become zero.[2] It is clear from equation (24) that as L is raised the number of non-zero dQ^+ falls and that on non-zero dQ^- rises. For instance, when L is assumed to be $^+4$ there are only 6 non-zero dQ^+; at the other extreme, when L is assumed to be -4 there are only 8 non-zero dQ^-.

Instead of the level of output Q we now have two series, namely dQ^+ and dQ^- in our regressions.[3] Since the coefficients of dQ^+ and dQ^- are *not* constrained to be equal they may produce a kink at Q_0. Otherwise our equations were the same as (16) and (17) and they will be referred to as (16c) and (17c). Since we have time series of dQ^+ and dQ^-

[1] The following four time trends were tried:

$$\underset{}{Q = 78.15 + \underset{(.03)}{.76t}} \qquad (\bar{R}^2 = .929)$$

$$Q = 78.85 + \underset{(.12)}{.73t} + \underset{(.0022)}{.0006t^2} \qquad (\bar{R}^2 = .928)$$

$$\log Q = 1.90 + \underset{(.0001)}{.0034t} \qquad (\bar{R}^2 = .924)$$

$$\log Q = 1.89 + \underset{(.0005)}{.0040t} - \underset{(.000009)}{.000012t^2} \qquad (\bar{R}^2 = .924)$$

[2] An example may help to clarify the procedure. Assume that we have five observations of dQ. The table shows that when $dQ < 0$ then $dQ^+ = 0$ and when $dQ > 0$ then $dQ^- = 0$.

dQ	dQ^+	dQ^-
−1	0	−1
+2	+2	0
+4	+4	0
−5	0	−5
0	0	0

[3] Since Q has a time trend which has been removed from dQ^+ and dQ^- our procedure must raise the coefficient of t in equations (16) and (17).

(one for each assumed L) there are 18 regressions all together. Table II contains the main results of these regressions; a^+ and a^- are the coefficients of dQ^+ and dQ^- in the employment demand function. The differences between the two original regression coefficients together with their standard errors are given under $\alpha(a^- - a^+)$.[1] A comparison of the values of $\alpha(a^- - a^+)$ with their standard errors show that the data have produced a kink which is significant at about the 5 per cent level when L ranges between 0 and +2. However, at all levels of L the kink is not of the type drawn in Figure 3. Invariably the positive deviations from Q_0 produce a *flatter* relationship between E^* and Q than do the negative deviations. Indeed, there is some slight evidence that the kink becomes more pronounced as Q_0 is assumed to rise. Thus far from supporting the proposition that $a^+ > a^-$ our evidence suggests fairly strongly that $a^- > a^+$. We must, therefore, consider the possible reasons for this observation. There are, at least, two:[2]

TABLE II

Coefficients of dQ^+ and dQ^- at Various Assumed Values of L

	Derived from Equation (16c)			Derived from Equation (17c)			Number of Observations	
L	a^+	a^-	$\alpha(a^- - a^+)$	a^+	a^-	$\alpha(a^- - a^+)$	Positive	Negative
+4	.169	.535	.0115 (.0169)	.206	.486	.0115 (.0152)	6	46
+3	.230	.549	.0122 (.0081)	.296	.494	.0082 (.0074)	10	42
+2	.219	.582	.0118 (.0051)	.269	.520	.0105 (.0047)	18	34
+1	.316	.601	.0093 (.0040)	.334	.534	.0083 (.0036)	21	30
0	.401	.607	.0067 (.0036)	.389	.539	.0063 (.0033)	28	24
−1	.444	.620	.0057 (.0036)	.419	.548	.0053 (.0033)	34	18
−2	.475	.625	.0048 (.0041)	.443	.550	.0044 (.0037)	40	12
−3	.484	.666	.0058 (.0053)	.452	.569	.0048 (.0048)	43	9
−4	.492	.736	.0077 (.0077)	.461	.591	.0053 (.0070)	44	8

[1] Let S_a and S_b stand for the standard errors of the regression coefficients a and b; if a and b are independent then the standard error of $(a - b)$ is $s_{a-b} = \sqrt{S_a^2 + S_b^2}$.

[2] A third possibility is that weight shifts occur in the output index. This proposition was examined by Kuh with U.S. data. In general it turned out to be not very important [12].

(i) There may be certain improvements in technical efficiency which are *necessarily* associated with a rise in output. In this case technological progress cannot be disentangled from the purely *ceteris paribus* relationship between E^* and Q.

(ii) In section I,1 we showed that a rise in E_s which is due to an increase in Q can lead to a rise in both E^* *and* U^* (see Figure 1). It is quite possible that positive deviations from Q_0 are accommodated largely by increases in U^* whilst negative deviations from Q_0 are reflected to a relatively larger extent by changes in E^*. If, for instance, E'_s corresponded to Q_0 then the line $D - A - E$ might illustrate the expansion path of E^* and U^* as output moved from a large negative deviation, through Q_0 to a large positive deviation. Expansion paths of the type illustrated by $D - A - E$ may produce the observed kink in the relationship between E^* and Q. Nor are such expansion paths highly implausible. Suppose that Q_0 is reached when employees and equipment are used for, say, 8 hours per day. An expansion in output might lead simply to a rise in hours (that is a rise in U^*) unless the expansion is so large that two shifts can be worked. On the other hand, a decline in output may lead to the stoppage of some machines and, the dismissal of some employees whilst the remaining employees continue to work an 8-hour day.[1]

6. *Conclusions*

This concludes our rather extensive discussion of the employment demand function. For reasons given in the introduction of this paper we decided to estimate this function within the framework of an adjustment mechanism. Even when fairly elaborate specifications of the employment demand function are used, the adjustment mechanism remains important.

The examination of our data has suggested that a quadratic time trend, the capital stock and the level of normal hours all appear to have a significant part to play in the employment demand function. Hence, the overall explained variance could be raised from .46 per cent in equation (6) to .83 per cent in equation (17). We regard this improvement in the fit as satisfactory. Moreover, the refinements of the employment demand function served to reduce the positive auto-correlation of the residuals substantially; the d statistic rose from .6 in equation (6) to 1.8 in equation (17). This gives us some confidence in the specification of the model.

We attempted to refine the employment demand function further, first, by introducing the *change* in output as an expectational variable and, second, by introducing a kink into the relationship between output and employment. Neither of these two operations achieved the expected result. The influence of changes in output is highly insignificant and, though there appears to be a kink in the relationship between output and employment it implies an increasing rather than a decreasing marginal product of labour. Hence, very considerable, apparently increasing, marginal returns to *ceteris paribus* increases in output have been observed.

III NON-SYMMETRIES AND CHANGES IN THE STRUCTURE OF THE MODEL.

In this part of the paper we shall be concerned with an examination of two propositions. According to the first, the adjustment process varies with the phase of the cycle. According to the second, far-reaching structural changes have occurred during the nineteen-fifties which may have affected both the parameters of the employment demand function and the adjustment process.

[1] Another possible explanation of the kink in the relationship between E^* and Q is that the adjustment process is incorrectly specified.

1. *Non-Symmetries in the Adjustment Process*

Throughout part II of this paper we assumed that the adjustment coefficient α did not vary with the phase of the cycle. We shall now admit of two adjustment coefficients: the first (α^+) refers to the cyclical upswing when $(E_t^* - E_{t-1})$ is positive and the second (α^-) to the cyclical downswing when $(E_t^* - E_{t-1})$ is negative.

It would appear that there are equally plausible arguments for supposing either that $\alpha^+ > \alpha^-$ or that $\alpha^- > \alpha^+$. For instance, α^+ should exceed α^- because the cost of overtime working is larger than the cost of short-time working. On the other hand, α^- should exceed α^+ because institutional limitations may make it virtually impossible to engage more employees at short notice and no such limitations apply to dismissals. The quantitative importance of these and similar arguments can only be assessed by looking at the evidence.

In order to see whether $\alpha^+ > \alpha^-$ or $\alpha^- > \alpha^+$ we fitted separate regressions to positive $\dot{E}$'s and negative $\dot{E}$'s.[1] These equations contained the same independent variables as equations (16) and (17) and they are, therefore, referred to as (16d) and (17d). The resulting α's, their differences and their standard errors are given in Table III.

TABLE III

Adjustment Coefficients for Positive and Negative Changes in Employment

Regression	α^+	α^-	$(\alpha^+ - \alpha^-)$
Equation (16d)	.254 (.042)	.088 (.095)	.166 (.104)
Equation (17d)	.334 (.066)	.402 (.123)	−.068 (.139)

Our results are slightly ambiguous because for equation (16d) $\alpha^+ > \alpha^-$ whilst for equation (17) $\alpha^- > \alpha^+$. However, the figures in the last column show that the differences between α^- and α^+ are quite insignificant.

We must conclude, therefore, that our evidence does not suggest that $\alpha^+ \neq \alpha^-$. Hence, the assumption of a constant α to which we adhered throughout part II of this paper, appears to be appropriate.[2] [3]

2. *Changes in the Structure of the Model*

It is sometimes asserted that through the nineteen-fifties both the parameters of the employment demand function and the adjustment process have changed. There are, of course, many possible reasons for such structural changes. We shall not enumerate any of them but go straight to the empirical evidence.

[1] There are 35 positive and 17 negative $\dot{E}$'s.

[2] Kuh [10] found evidence that in the United States α^- seems to exceed α^+.

[3] Ball and St. Cyr [2] found that the adjustment coefficients differed significantly between industries Industries with high unemployment tended to have faster adjustments (that is, higher α's) than industries with low unemployment. Though the aggregate α does not appear to differ between upswings and downswings, Ball and St. Cyr's results suggest that it may vary with the level of unemployment.

We attempted to test for differences in the structure by dividing the total sample period into two halves. There are 26 observations in each sub-period, the first covers 1950, I to 1956, II and the second 1956, III to 1962, IV. The sub-periods have roughly similar cyclical patterns. Both start with a minor boom (1951 and 1957); this is followed by a major recession (1952 and1958); then there is an upswing which reaches a peak in 1955 and 1960; finally there is a fairly severe recession.

We fitted four regressions, two have the same independent variables as equation (16) and the other two correspond to equation (17). Hence, we shall refer to them as equations (16e) and (17e). The regression coefficients, their differences, their standard errors, the $\bar{R}^2$'s and the d's are presented in Table IV.

We subjected our two sets of equations to the standard test of equality between sets of coefficients.[1] For the appropriate degrees of freedom the F ratio would have to exceed 2.34 if the differences between the regressions for the two sub-periods were to be significant

TABLE IV

Equations (16*e*) and (17*e*)

Independent Variables	Equations (16e)			Equations (17e)		
	(i) 1950-1956	(ii) 1956-1962	Difference	(i) 1950-1956	(ii) 1956-1962	Difference
Q_t	.136 (.024)	.203 (.047)	—.067 (.053)	.170 (.021)	.241 (.025)	—.071 (.033)
t	—.034 (.047)	—.061 (.267)	.027 (.271)	.039 (.024)	.122 (.103)	—.083 (.106)
t^2	.00051 (.00131)	—.00059 (.00327)	.00110 (.00352)	—.00075 (.00120)	—.00443 (.00161)	.00368 (.00201)
R_t	—.0489 (.0282)	.0067 (.0334)	—.0556 (.0437)			
H_t				.799 (1.881)	—.592 (.208)	1.391 (1.893)
E_{t-1}	—.349 (.067)	—.256 (.067)	—.093 (.095)	—.435 (.062)	—.496 (.098)	.061 (.116)
$\bar{R}^2$	.82	.80		.79	.86	
d	2.11	1.83		2.17	2.25	

at the 5 per cent level. The actual F ratios turned out to be 2.26 for equations (16e) and 1.84 for equations (17e). Thus equations (16e) show a break in the relationship which is nearly significant at the 5 per cent level.

[1] See Johnston [9], p. 136.

Turning now to the individual regression coefficients we find that only one seems to differ significantly between the two periods: it is the coefficient of Q_t in equations (17e). which seems to have risen significantly between the two periods. This result indicates that the relationship between E^* and Q has become slightly steeper in recent years, so that productivity gains associated with *ceteris paribus* increases in output were smaller in the second than in the first period.

IV SOME APPLICATIONS OF THE RESULTS.

In this part of the paper we intend to give an indication of the quantitative importance of some of our results. First, we shall argue that serious errors may be committed if the dynamic adjustment process is neglected and, second, we shall deal with the implications of the underlying employment demand function. We shall confine our attention to the results of equations (17) and (17a).

1. *The Importance of the Dynamic Adjustment Process*

We have already presented evidence according to which the dynamic adjustment process is a very significant statistical relationship. We must now assess its quantitative importance. In Figure 4 E_t^* (derived from equation (17e)) and E_t have been plotted. A

FIGURE 4

comparison of the two series shows, first, that at times they differ quite substantially and, second, that the differences between them are non-random and have a cyclical pattern.[1] Hence, if for some purpose, we approximated E_t^* by E_t we would introduce a systematic error into our calculations.

[1] On average the positive differences between $E_t{}^*$ and E_t amount to .8 per cent and the negative differences to .5 per cent.

As an example of the kind of error that may occur, consider the index of productivity $\frac{Q}{E}$. As an index of the *underlying* output per man it will be subject to systematic errors which, over the business cycle, vary between approximately $-.8$ and $+1.5$ per cent. This error is large when compared with the annual average rise in output per man of about 2.5 per cent. Hence we conclude that $\frac{Q}{E}$ will be dominated by cyclical factors which may prevent us from analysing the *underlying* series of output per man.

Neglect of the dynamic adjustment process may have led to another kind of error. Many empirical investigators and short-term forecasters have placed great emphasis on the unemployment ratio as an index of the pressure on capacity in the economy. Since it ignores the difference between E_t^* and E_t the unemployment ratio may, however, not be a particularly useful measure of the pressure of demand during the business cycle. As an example, consider the fourth quarter of 1959 when the unemployment ratio stood at 1.99 per cent and E_t^* exceeded E_t by 1.4 per cent. If output and normal hours had not changed, but E_t had adjusted to E_t^* then unemployment would have fallen to approximately .6 per cent.[1] In this case, therefore, the relatively high unemployment figure was quite a misleading indicator of the pressure of demand.

The examples of both the productivity index and the unemployment ratio have shown that the quantitative importance of the dynamic adjustment process is likely to be substantial and that serious systematic errors may arise when it is neglected.

2. *The Underlying Rate of Productivity Growth*

In part II we examined the separate influences of the various exogenous variables upon the underlying employment demand. We must now see what would happen if several variables moved together.

To start with we must make an assumption about the movement of output. We shall abstract from its cyclical variations and assume that it is equal to its trend value (Q_{et}) given by equation (23). When (23) is substituted for Q_t in equation (17a) the latter becomes:

$$E_t^* = 173.639 + .474t - .0059t^2 - .815H_t. \tag{17f}$$

The level of labour productivity is now defined as

$$P_{et} = \frac{Q_{et}}{E_t^*} \tag{25}$$

which can be computed from (23) and (17f).

TABLE V

t	Q_{et}	P_{et}	$\frac{dE_t^*}{dt}$	$\frac{dP_{et}}{dt}$	$\frac{1}{P_{et}}\frac{dP_{et}}{dt}$
1	78.910	85.253	.4622	.399	.468
52	117.859	116.955	$-.1401$	.920	.787

[1] We assume that the rest of the economy behaved like the manufacturing sector.

Table V shows the values of some of our variables at $t = 1$ and $t = 52$ on the assumption that H_t remained unchanged at 100. The calculations show that the rate of change in output per man would have accelerated very markedly throughout the sample period. Thus the percentage rate of change of productivity growth would have risen from .468 to .787 per quarter and the arithmetic rate of change from .399 to .920.

Since we have not observed such a rapid acceleration in the underlying rate of growth of labour productivity the assumption of a constant level of normal hours must be crucial. Let us, therefore, look at the relationship between P_{et} and H_t. The values of the derivative $\frac{dP_{et}}{dH_t}$ are given in Table VI for the beginning and the end of the period. They show that a given reduction in H had a larger negative influence upon P_{et} at the end than at the beginning of the sample period; but the difference between the two estimates is not very large and, hence, we shall use a value of .8 for all t.

TABLE VI

t	H_t	$\frac{dP_{et}}{dH_t}$
1	100	—.75
52	94.9	—.87

The maximum observed quarterly fall in H_t of .7 points may have reduced P_{et} by as much as .56 points and this value is large compared with the quarterly rise of .399 to .920 generated by Q and the quadratic time trend. Between 1960 and the end of 1962 normal hours fell by over 4 points and this may have reduced labour productivity by as much as 3.2 points.

In this section we analysed the long-run growth of output per man. In order to abstract from cyclical phenomena we examined what would have happened if output had grown at its trend rate. We discovered that there is a fairly marked tendency for the productivity growth rate to accelerate but that this acceleration was offset by falling normal hours.

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APPENDIX A

Throughout this paper we used simple linear regressions which related employment to various exogenous variables. We ran analogous regressions for men-hours (E') and the logarithms of men and man-hours. The equations which correspond to (16) and (17) turned out as follows:[1]

$$\text{(Ai)}\quad \Delta \log E = \underset{(.062)}{.291} + \underset{(.016)}{.164} \log Q_t - \underset{(.00015)}{.00029}t - \underset{(.00000173)}{.00000037}t^2 + \underset{(.00210)}{.00168} \log K'_t - \underset{(.039)}{.307} \log E_{t-1} \qquad \begin{pmatrix}\bar{R}^2 = .72\\ d = 1.30\end{pmatrix}$$

$$\text{(Aii)}\quad \Delta \log E = \underset{(.227)}{1.096} + \underset{(.014)}{.182} \log Q_t + \underset{(.00011)}{.00013}t - \underset{(.0000021)}{.0000087}t^2 - \underset{(.086)}{.319} \log H_t - \underset{(.044)}{.408} \log E_{t-1} \qquad \begin{pmatrix}\bar{R}^2 = .78\\ d = 1.59\end{pmatrix}$$

$$\text{(Aiii)}\quad \dot{E}' = \underset{(4.52)}{33.39} + \underset{(.035)}{.325}Q_t + \underset{(.0368)}{.0995}t - \underset{(.0006)}{.0042}t^2 - \underset{(.0143)}{.0170}R_t - \underset{(.064)}{.642}E_{t-1} \qquad \begin{pmatrix}\bar{R}^2 = .73\\ d = 1.70\end{pmatrix}$$

$$\text{(Aiv)}\quad \dot{E}' = \underset{(19.16)}{69.01} + \underset{(.036)}{.358}Q_t + \underset{(.0514)}{.1631}t - \underset{(.00116)}{.00613}t^2 - \underset{(.164)}{.328}H_t - \underset{(.072)}{.706}E_{t-1} \qquad \begin{pmatrix}\bar{R}^2 = .74\\ d = 1.80\end{pmatrix}$$

$$\text{(Av)}\quad \Delta \log E' = \underset{(.101)}{.614} + \underset{(.041)}{.291} \log Q_t + \underset{(.00033)}{.00056}t - \underset{(.000003)}{.000017}t^2 - \underset{(.00488)}{.00507} \log K'_t - \underset{(.077)}{.590} \log E_{t-1} \qquad \begin{pmatrix}\bar{R}^2 = .73\\ d = 1.66\end{pmatrix}$$

$$\text{(Avi)}\quad \Delta \log E' = \underset{(.396)}{1.455} + \underset{(.035)}{.341} \log Q_t + \underset{(.00023)}{.00061}t - \underset{(.000005)}{.000024}t^2 - \underset{(.167)}{.347} \log H_t - \underset{(.073)}{.717} \log E_{t-1} \qquad \begin{pmatrix}\bar{R}^2 = .74\\ d = 1.77\end{pmatrix}$$

[1] Note that since R_t has negative and positive observations it had to be replaced by K'_t in equations (Ai) and (Av).

APPENDIX B

Output, Employment, Factory Completions and Normal Hours in Manufacturing. (Seasonally Adjusted)

		Output 1958=100	Employment 1958=100	Factory Completions (m. sq. ft.)	Normal hours* 1950=10
1949,	IV	75.3	91.3	4.6	100.0
1950,	I	79.6	92.1	5.7	100.0
	II	80.2	92.8	7.9	100.0
	III	83.6	93.1	6.8	100.0
	IV	83.4	93.9	4.6	100.0
1951,	I	83.8	94.8	5.7	100.0
	II	85.8	95.3	10.0	100.0
	III	87.2	95.7	8.7	100.0
	IV	83.4	95.5	9.4	100.0
1952,	I	83.6	95.4	9.5	100.0
	II	80.2	94.6	7.5	100.0
	III	79.6	94.1	6.6	100.0
	IV	81.5	94.4	6.1	100.0
1953,	I	83.6	94.7	10.0	99.9
	II	85.4	95.4	6.3	99.9
	III	87.8	96.2	6.2	99.9
	IV	89.8	96.9	8.8	99.9
1954,	I	90.6	97.1	9.7	99.9
	II	93.1	97.8	12.0	99.9
	III	94.9	98.8	8.4	99.9
	IV	95.9	99.6	8.9	99.9
1955,	I	97.5	100.1	8.6	99.8
	II	99.1	100.6	9.9	99.8
	III	100.3	101.1	11.3	99.8
	IV	102.0	101.7	11.4	99.8
1956,	I	99.4	101.8	11.2	99.8
	II	99.2	101.5	11.0	99.8
	III	98.8	101.2	13.4	99.8
	IV	98.8	101.0	12.6	99.8
1957,	I	100.4	101.0	14.0	99.7
	II	101.4	101.3	13.6	99.7
	III	102.8	101.8	13.5	99.7
	IV	100.7	101.5	12.3	99.6

		Output 1958=100	Employment 1958=100	Factory Completions (m. sq. ft.)	Normal hours* 1950=10
1958,	I	101.0	100.9	12.9	99.6
	II	99.6	100.3	12.3	99.5
	III	99.3	99.7	10.8	99.5
	IV	100.1	99.1	13.4	99.5
1959,	I	101.1	99.1	7.8	99.5
	II	105.0	99.8	9.6	99.5
	III	106.8	101.0	10.1	99.4
	IV	111.1	101.9	8.2	99.4
1960,	I	113.6	102.7	11.6	99.1
	II	114.8	103.8	10.2	98.0
	III	115.1	104.9	11.6	97.7
	IV	114.5	104.9	14.3	97.0
1961,	I	115.1	105.0	13.2	96.3
	II	115.4	105.6	12.7	96.0
	III	115.1	105.7	12.7	95.6
	IV	113.9	105.3	13.3	95.2
1962,	I	113.8	105.0	9.2	95.0
	II	115.3	104.8	10.1	95.0
	III	116.3	104.7	11.9	95.0
	IV	115.7	103.9	11.7	94.9
1963,	I	114.5	103.2	7.7	94.9
	II	117.6	103.1		94.9
	III	122.0	103.4		94.9
	IV	124.8	103.6		94.7

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