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The Elasticity of Substitution Between Capital and Labour in the US Economy: A Meta-Regression Analysis*

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Abstract

Despite extensive research, there is no agreement on the value of the elasticity of substitution between capital and labour at the aggregate or the industrial level. Utilizing 2,419 estimates from 77 studies published between 1961 and 2017, this paper provides the first meta-regression analysis for the US economy. We show that the heterogeneity in previously reported estimates is driven primarily by modelling decisions for technological dynamics. Throughout the analysis, the hypothesis of a Cobb–Douglas production function is rejected. Based on our meta-regression sample, we estimate a long-run meta-elasticity for the aggregate economy in the range of 0.45–0.87. Most industrial estimates do not deviate significantly from the estimate for the aggregate economy.

I. Introduction

The elasticity of substitution between capital and labour, σ, is an essential concept in macroeconomics. In theories of economic growth, the value of σ is crucial for a multitude of issues, including the diminishment of marginal products (Brown, 1966), the speed of convergence to the steady state (Turnovsky, 2002) or the role of technological change (Acemoglu, 2003). As already shown by Solow (1956) and Pitchford (1960), perpetual growth is possible even in the absence of technical progress if σ is sufficiently large. Moreover, the substitutability between capital and labour also plays a key role for the functional distribution of income. Since the seminal contribution of Hicks (1932), it is well known that changes in relative factor endowments affect the distribution of income between capital and labour if σ differs from unity. For example, Piketty's (2014) explanation

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of the observed decline in the labour share of income crucially depends on the assumption that $\sigma > 1$ (Piketty and Zucman, 2014; Acemoglu and Robinson, 2015).

Due to its relevance, consistent evidence on plausible values of σ would be valuable for researchers and policy advisers. However, since the introduction of the constant elasticity of substitution (CES) production function by Arrow *et al.* (1961), no consensus has emerged in the empirical literature. For the US economy, Chirinko's (2008) survey reports values above, below and at unity. More recently, Grossman *et al.* (2017, pp. 1295) state that '[t]he size of the elasticity of substitution is much debated and still controversial'.

Past research has identified several regularities underlying the heterogeneity in estimation results. For instance, Lucas (1969) notes that early time series studies of the USA typically reject the Cobb–Douglas assumption, whereas cross-sectional estimates tend to support a unitary elasticity. Another regularity appears to be present with respect to the choice of the estimation equation. Over a multitude of studies (e.g. Dhrymes and Zarembka, 1970; Kalt, 1978; Young, 2013), the use of the first-order condition for labour yields estimates exceeding those obtained by the use of the first-order condition for capital. In addition, Antràs (2004) reveals a regularity with respect to the treatment of technological change. Based on theoretical and empirical evidence he demonstrates that the assumption of Hicks-neutrality biases estimates towards unity in the presence of non-neutral technological change and roughly constant factor income shares. As will be outlined below, this list of conjectures can be extended considerably.

The contribution of this paper is the first assessment of heterogeneity in estimates of the elasticity of substitution between capital and labour in the US economy both at the aggregate and the industrial level within a meta-regression framework.[1] In addition to analyzing the effects of the different specification choices, we estimate meta-elasticities of substitution based on our collected data. Conditional on these meta-estimates, we provide evidence that for the economy as a whole, σ most likely falls in the range of 0.45–0.87. Thus, the aggregated empirical evidence suggests that the US economy is most likely not well represented by a Cobb–Douglas production function, which implies $\sigma = 1$. Furthermore, we find that heterogeneity in previously reported estimates is mainly driven by different specifications of technological dynamics.

The remainder of the paper is structured as follows. Section II briefly reviews central properties of the CES production function. In section III, we introduce our search strategy, provide an initial overview of the collected elasticity estimates and explore various possible sources of heterogeneity. The actual meta-regression analysis is conducted in section IV, where we first describe the data set and our empirical strategy. Subsequently, estimation results are presented, followed by robustness checks and extensions. Section V concludes.

[1] Although still less applied than in other disciplines, meta-regressions have become increasingly popular in economics. For example, Lichter, Peichl and Siegloch (2015) analyze the own-wage elasticity of labour demand and Doucouliagos and Ulubasoglu (2008) focus on democracy and economic growth. For excellent overviews of meta-(regression) analysis methods see Feld and Heckemeyer (2011) and Stanley and Doucouliagos (2012). Meta-analyses based on observational data are usually subject to the limitation that different studies may draw on the same data sources. This results in an informational overlap between the estimates reported in these studies. Since these overlaps to some extent also exist in our data set, our analysis shares this limitation with other meta-analyses in economics. However, there is variation in the data sources used by the studies included in our sample. Furthermore, these studies cover different time periods and different levels of aggregation (firm/industry/country level). Against this background, applying meta-analytic techniques can provide informative insights on the heterogeneity in estimates of the elasticity of substitution.

II. CES production function

In preparation for the following discussion of the potential sources of heterogeneity, this section briefly introduces the CES production function. Its central parameter is the elasticity of substitution between capital K and labour L, defined independently by Hicks (1932) and Robinson (1933):

$$\sigma = \frac{d(K/L)/(K/L)}{d(F_L/F_K)/(F_L/F_K)} \tag{1}$$

where output Y is produced by a linear homogeneous production function, $Y = F(K, L)$. $F_L \equiv \partial Y/\partial L$ and $F_K \equiv \partial Y/\partial K$ denote the marginal productivities of the inputs. Following equation (1), the elasticity of substitution can be regarded as the percentage change in the capital-labour ratio due to a percentage change in the ratio of the marginal products of inputs along a given isoquant curve.[2] Under certain conditions the standard Arrow *et al.* (1961) specification of the CES function can be derived from definition (1).[3] Allowing for the possibility of factor-augmenting technological change as in David and Van de Klundert (1965), a more general variant of the CES production function is

$$Y_t = C[\pi(A_t^K K_t)^{\frac{\sigma-1}{\sigma}} + (1-\pi)(A_t^L L_t)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} \tag{2}$$

where C is an (Hicks-neutral) 'efficiency' parameter and $0 < \pi < 1$ refers to a 'distribution' parameter that determines the relative importance of capital and labour in production. The positive coefficients A_t^K and A_t^L capture the level of efficiency of capital and labour inputs respectively. Variations over time t are regarded as capital- and labour-augmenting technological change. Like all standard CES functions, (2) nests a Cobb–Douglas function for $\sigma \to 1$, a Leontief function for $\sigma \to 0$ and a von Neumann production function for $\sigma \to \infty$.[4]

III. The meta-sample and sources of heterogeneity

To construct a comprehensive database, we began our search process by examining several literature reviews to identify relevant studies.[5] Based on the identified literature, we selected potential keywords to conduct different search queries to capture all remaining relevant studies. As sources of peer-reviewed publications, Web of Science and EBSCOhost were examined from the years 1961 to 2017. Search terms included, among others, 'capital', 'labor'/'labour', 'elasticity of substitution' and 'estimation'. To obtain additional literature sources (working papers, books and dissertations), we also conducted a Google Scholar search. We had to find an appropriate balance between an enhanced meta-sample that improves the statistical power of our estimations and high comparability across studies. To reconcile these two aims, we selected studies fulfilling the following criteria:

[2] Since its introduction, a multitude of variations and generalizations of the elasticity of substitution have been developed. Stern (2011) presents a useful classification scheme for the various definitions and discusses how they are related to one another.

[3] See de La Grandville (2017) for a derivation.

[4] In recent years, some studies make use of a 'normalized' variant of the CES function for comparative statics (see Klump, McAdam and Willman, 2012). For a critical discussion see Temple (2012).

[5] These are Nerlove (1967), Caddy (1976), Kalt (1978), Chirinko (2002), Klump, McAdam and Willman (2007b), Chirinko (2008), León-Ledesma, McAdam and Willman (2010) and Klump *et al.* (2012).

(i) The estimates were conducted for the US economy either at the aggregate or the industrial level.
(ii) The estimates attribute homogeneity within each of the two production factors.[6]
(iii) The estimation equation of the study is derived from a CES production function specification.[7]

To complete our search process, manual searches were performed to identify additional studies using the reference list of each selected study. We considered prior versions of each study and included diverging estimates. Furthermore, some of the estimates reported by Dhrymes (1965) are omitted due to their correction in Dhrymes and Zarembka (1970). Likewise, the estimates in Moroney (1970) are replaced by the corrected estimates presented in Lovell and Moroney (1973). The resulting meta-data comprise 2,419 observations gathered from 77 studies published between 1961 and 2017. Summary information for each study can be found in the online appendix. The search was conducted between February and December 2017.

First glance at the collected elasticities

Figure 1 depicts the distribution of all collected estimates in the form of open-ended histograms. The upper graph shows estimates which have been conducted at the manufacturing or a higher aggregation level. The lower graph shows estimates at the industry level. In both graphs, the vast majority of elasticities cluster between 0 and slightly above 1. Between these values, no clear pattern towards a specific value is observable. Many values scatter around 0.5 for the USA as a whole, but there is also a dominant peak between 0.9 and 1. The distribution of estimates on the industry level is less concentrated and almost flat between 0 and 1. Only a small fraction exceeds the value of 2 in both graphs and almost no estimate is negative.[8]

Combined with the histogram, Figure 1 also shows the individual estimates in a so-called 'funnel plot'. This illustrates in greater detail the relationship between the estimates of σ and the inverse of their standard errors ($1/se(\tilde{\sigma})$), a common measure of precision in meta-studies. As is readily observed, the estimates in the range between 0 and 1 are the most precise.[9] The absence of a clear funnel shape indicates study heterogeneity. A peak can be identified around a value of 1, indicating a Cobb–Douglas production function. Precision is also slightly higher in the neighbourhood of 0.4. Most interestingly, there is a considerable cluster of high-precision estimates slightly above zero.[10] A simple average of all 853 estimates for the aggregate economy suggests an elasticity of 0.54, a value slightly above the first peak in the histogram–funnel plot. Assigning more weight to more precisely estimated

[6] Studies that adopt a capital-skill complementarity framework, where the elasticity of substitution is estimated separately for different skill groups, as in Griliches (1969) or more recently in Krusell *et al.* (2000), are excluded.

[7] Thus, we excluded estimates based on alternative concepts like translog or VES production functions. This reflects the dominant role of the CES production function in the literature.

[8] Although negative estimates are theoretically implausible, they can still occur. Imagine the production function would be truly of Leontief form, that is, $\sigma = 0$. In this case, we should observe estimates below the true value of 0 as well as above due to sampling variability.

[9] Only estimates with a precision smaller than 200 are shown for readability.

[10] The same graph is shown for all industries individually in Figure A1 in the appendix. As can be seen, multiple industries show a precision peak close to zero.

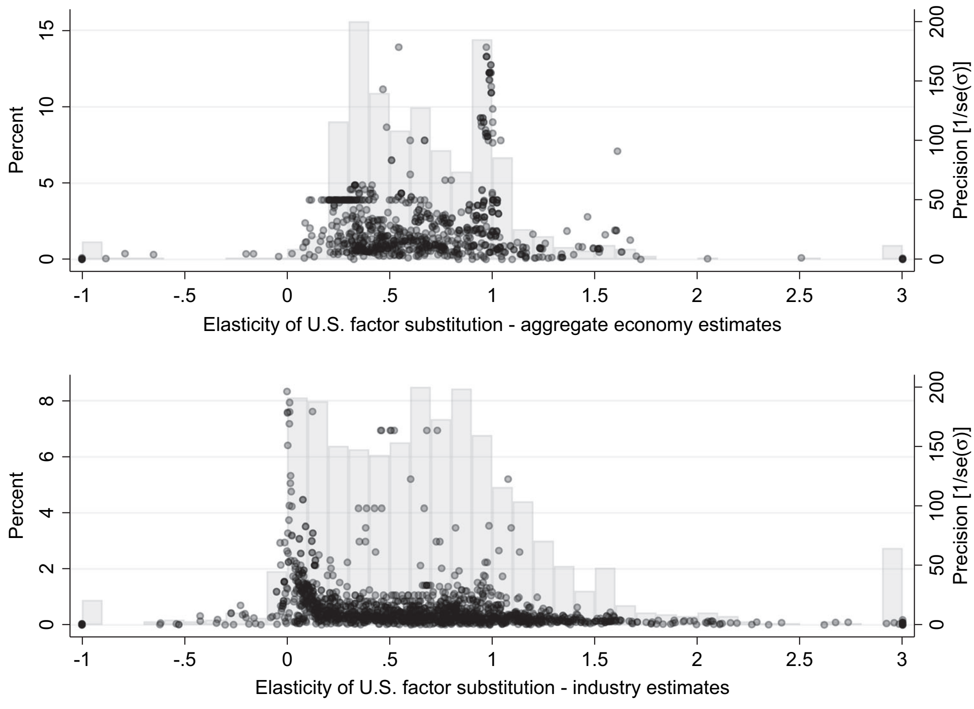

Figure 1 Open-ended histogram and funnel plot, depicting collected estimates for the elasticity of factor substitution on the aggregate as well as the industry level

values shifts the mean to 0.77 (inverse standard error weighted). As the funnel plot is not symmetrically shaped and excess variation clouds the picture, inquiry beyond traditional surveys is necessary.[11] To explain why the estimates differ, it is important to collect more information on the study designs. We will investigate below which of these design choices crucially affect the estimation of the elasticity of substitution between capital and labour.

Sources of heterogeneity

To establish a comprehensive data sample, we identify the following likely sources of heterogeneity in the estimates of σ, which we summarize in the following categories: (i) functional form of the estimation equation, (ii) technological dynamics, (iii) estimation characteristics, (iv) data characteristics and (v) general study characteristics.

Functional form of the estimation equation

To estimate the CES production function, various empirical strategies have been developed. The following analysis distinguishes (i) all variants of first-order conditions of profit maximization.[12] Additionally, we include (ii) all types of two- or three-equation systems, as for instance applied in Bodkin and Klein (1967), Klump, McAdam and Willman (2007a)

[11] Instead of plotting every single estimate, one could also just rely on the authors' preferred estimate. This is not feasible in our case. Usually authors do not indicate their preferred estimate.

[12] A derivation and complete assembly of all FOC variants considered in the meta-regression analysis can be found in the online appendix.

and León-Ledesma, McAdam and Willman (2015), (iii) a direct (nonlinear) estimation of the CES production function and (iv) linear approximations, including the Kmenta (1967) approximation.[13] Some studies (e.g. Takayama, 1974; Young and Cen, 2007) apply a FOC specification in growth rates, rather than in levels, for inputs and factor prices to estimate σ. All estimates based on such an approach are captured by a growth rates dummy. Although the majority of studies maintain the assumption of purely competitive product and factor markets, some studies allow for a potential mark-up over factor costs. Thus, a mark-up dummy distinguishes all estimates that both freely estimate a time-varying, input-specific (e.g. Raurich, Sala and Sorolla, 2012) or a time- and factor-averaged (e.g. Klump, McAdam and Willman, 2004) mark-up as well as estimates that apply a predetermined positive value (e.g. León-Ledesma *et al.*, 2010).

Technological dynamics

As the influential contribution by Antràs (2004) reveals, an important aspect of estimating σ is the treatment of technological dynamics. One can distinguish between estimations that neglect technological progress and those that account for changes in technology parameters. The latter, in turn, can be categorized according to the specified type(s) of technological change, that is, Hicks-neutral, capital-augmenting, labour-augmenting or some combination thereof, and the specific form of technological dynamics, that is, a constant growth rate or flexible dynamics modelled through a Box and Cox (1964) transformation.[14] This specification underlies 1% of all estimates. While the 'constant growth rate' specification of technological dynamics is the most frequently used in the literature, there are alternative specifications, which are coded as 'other dynamics' in the following. Therefore, in the following analysis, we distinguish between (i) factor-augmenting (i.e. capital-augmenting, labour-augmenting or both) technological change assuming a constant growth rate, (ii) factor-augmenting specifications relying on the Box–Cox transformation, (iii) other factor-augmenting dynamics, (iv) Hicks-neutral specifications assuming a constant growth rate and (v) estimations neglecting technological change.[15] Because Hicks-neutral and factor-augmenting specifications are econometrically equivalent in case of the FOCs, we consider those estimations to capture factor-augmenting technological change, even if the corresponding paper draws on a Hicks-neutral model.

Estimation characteristics

As the majority of the studies included in our data set use OLS or nonlinear least squares (NLLS), we distinguish between least squares estimates and estimates obtained by applying other methods, such as generalized method of moments (GMM) or maximum likelihood (ML).

Another problem is the potential endogeneity of regressors. For instance, the first-order conditions of profit maximization can be interpreted as describing firms' aggregate demand

[13] Note that we also treat a simultaneous estimation of two FOCs, as applied in Kalt (1978), among others, as an equation system.

[14] The assumption of a constant growth rate of technological efficiency is typically chosen to circumvent problems related to the Diamond, McFadden and Rodriguez (1978) impossibility theorem. An application and introduction of the Box–Cox transformation of technological change can be found in Klump *et al.* (2007a).

[15] Note that we did not observe Hicks-neutral specifications employing the Box–Cox transformation.

for capital and labour respectively. Estimations relying on FOC equations can, therefore, be subject to simultaneous equation bias unless exogenous variables affecting supply are used in the estimation procedure (Hausman, 1978; Antràs, 2004). Typically, such endogeneity problems are addressed by applying instrumental variable (IV) regression. Our meta-regression analysis estimates the effect of endogeneity correction by including a variable capturing whether IV techniques were applied. However, a condition required for consistency of the IV estimator is that there is no correlation between the instruments and the error term. Otherwise, the IV estimator is inconsistent. Another known problem of the IV estimator is that it may be biased towards the OLS estimator in finite samples when instruments are weak. In case of both weak instruments and correlation between the instruments and the error term, the IV estimator may even be more inconsistent than the OLS estimator (Cameron and Trivedi, 2005). Since the exogeneity of the instruments used for the IV estimations included in our sample cannot be assessed, differences between results relying on IV estimation and other estimation approaches should be interpreted with caution.

From a theoretical perspective, the firms' first-order conditions for profit maximization refer to long-run relationships between factor inputs and factor prices. In the short run, however, firms are likely to face adjustment frictions and hence cannot be expected to immediately respond to changes in factor prices according to these equations. Consequently, one should expect that the elasticity of substitution between input factors is lower in the short run. Turning to the econometric perspective, researchers should, therefore, be aware of the gap between the long-run nature of the theoretical concept and the short-run nature of the data usually available for estimation. Several approaches to solve this problem have been proposed in the literature, including convex adjustment cost models, cointegration techniques (Caballero, 1994) or the use of a low-pass filter (Chirinko and Mallick, 2017). Following Chirinko (2008), we, therefore, distinguish among (i) estimates of the short-run elasticity of substitution, typically derived by explicitly modelling frictions, (ii) long-run estimates relying on cointegration models, low-pass filtering or time averaged data and (iii) 'inconsistent' approaches seeking to obtain the long-run elasticity at the theoretical level (e.g. by the use of unadjusted first-order conditions) but relying on unadjusted (short-run) data.

Data characteristics

Our sample comprises estimates based on cross-sectional, time series and panel data. Furthermore, some regressions rely on aggregate data for the US economy, whereas others are located at the industry or firm level. We, therefore, code whether an estimation relies on (i) cross-sectional, (ii) time series or (iii) panel data. Another variable captures whether the evidence is based on (i) country-level, (ii) industry-level or (iii) firm-level data.

The theoretical and empirical literature on economic growth stresses the relevance of human capital accumulation. Therefore, some empirical studies adopt approaches that 'adjust' labour input for human capital instead of relying on indicators of raw labour. Against this background, we distinguish between estimations based on any type of quality-adjusted labour and those based on unadjusted labour input.[16]

[16] However, note that our data set does not include estimates of the elasticity of substitution between capital and specific skill groups of workers.

General study characteristics

To account for differences in the type of publication, we include dummies for peer-reviewed journal articles, working papers and monographs (including books, handbooks and dissertations).

IV. Meta-regression analysis

The literature in general and the Monte Carlo studies by León-Ledesma *et al.* (2010, 2015) in particular provide some evidence on the characteristics of a state-of-the-art estimation. Their simulations show that some model specifications lead to over- or underestimations of the underlying parameter. These insights allow us to define reference categories for each variable in the regressions. The constant of such a meta-regression model can be interpreted as the meta-estimate of σ, which one can expect to obtain with the reference specification.[17] Before we discuss our method in detail, we first describe our sample and the choice of suitable reference categories for estimation.

Descriptive statistics and reference categories

Table 1 summarizes the distributional properties of our variables that are supposed to explain the differences in the estimates of σ. The data set contains estimates from 77 studies with an average of 31 estimates per study. Journal articles form our reference category because it is assumed that peer review promotes the publication of high-quality estimates. They represent 63% of the observations in the sample. The second largest category is working papers with 28%, while estimates from monographs account for only 9% of all observations.

Regarding the choice of the estimation equation, 14% of all estimates derive from equation systems. Based on Monte Carlo simulations, León-Ledesma *et al.* (2010) provide evidence that the simultaneous estimation of the production function with one or both FOCs generally yields good estimates of σ. Contradictory evidence is obtained by Stewart and Li (2018). The authors note that the equation system approach does not appear to provide a framework that overcomes the Diamond–McFadden–Rodriguez non-identification result (Diamond *et al.*, 1978) for Canadian data. Bearing this result in mind, we focus on estimates of σ for the US economy using 'equation system' as our reference category. Production function estimations represent 1% of the sample, whereas most studies opt for one of the several versions of the first-order conditions.

In the possible case of imperfect factor market competition, it is necessary to control for mark-ups to avoid bias, and, thus, estimates incorporating a mark-up in the model represent our reference specification. Estimating in growth rates is likely to induce approximation error, as those estimation equations are based on time derivatives, whereas real-world data are 'discrete' in nature. Therefore, estimation in levels is chosen as the reference category.

With respect to technological dynamics, the Box–Cox transformation is the most flexible approach and, therefore, chosen as the reference category. Only 1% of all estimations made

[17] Technically, such estimates are 'within-sample predicted values of the dependent variable under a particular set of conditions' (Nelson and Kennedy, 2008, pp. 346). This requires all regressors to be categorical variables, as will be shown below.

TABLE 1

Descriptive statistics (N = 2.419)

	Mean	*SD*	*Min*	*Max*
Estimates per study	31.01	44.89	1	250
Journal article estimates (Ref.)	0.63	0.48	0	1
Monograph estimates	0.09	0.29	0	1
Working paper estimates	0.28	0.45	0	1
Equation system (Ref.)	0.14	0.34	0	1
Direct estimation	0.01	0.09	0	1
Linear approx.	0.03	0.18	0	1
FOC capital	0.14	0.35	0	1
FOC capital-labour combined	0.24	0.43	0	1
FOC labour	0.22	0.42	0	1
Reverse FOC capital	0.01	0.09	0	1
Reverse FOC capital-labour combined	0.02	0.15	0	1
Reverse FOC labour	0.02	0.16	0	1
Estimation in factor shares	0.16	0.36	0	1
Mark-up (Ref.)	0.02	0.14	0	1
No Mark-up	0.98	0.14	0	1
Estimations in levels (Ref.)	0.78	0.41	0	1
Estimations in growth rates	0.22	0.41	0	1
Factor-augmenting, Box–Cox (Ref.)	0.01	0.10	0	1
Factor-augmenting, constant growth	0.39	0.49	0	1
Factor-augmenting, other	0.11	0.32	0	1
Hicks-neutral, constant growth	0.02	0.13	0	1
No technological dynamics	0.47	0.50	0	1
Long-run σ (Ref.)	0.16	0.36	0	1
Short-run σ	0.07	0.25	0	1
Theoret. long-run/emp. short-run σ	0.78	0.42	0	1
Least squares estimation (Ref.)	0.90	0.30	0	1
Other estimation method	0.10	0.30	0	1
IV estimations (Ref.)	0.20	0.40	0	1
Non-IV estimations	0.80	0.40	0	1
Country (Ref.)	0.20	0.40	0	1
Industry level	0.67	0.47	0	1
Firm level	0.13	0.33	0	1
Time series (Ref.)	0.54	0.50	0	1
Panel	0.11	0.31	0	1
Cross section	0.34	0.48	0	1
Quality-adjusted labour (Ref.)	0.25	0.43	0	1
Unadj. labour	0.75	0.43	0	1

use of it, whereas 39% assume factor-augmenting technological change with a constant growth rate. Finally, 47% do not specify any technological dynamics at all.

In most cases, the theoretical long-run model conflicts with the empirical treatment of the data. Approximately 78% of all estimates fall into this category. Their results should be in between the short- and long-run values in the strict sense. The reference category covers all studies with a theoretical and empirical model that feature a long-run elasticity

(16% of the observations). We choose 'long-run' estimates of σ as the reference category, as most of the literature is interested in long-run relationships.[18]

Nearly all studies use some sort of least squares estimation, which serves as reference. A few studies reporting many estimates employ other estimation methods.[19] Instrumental variable (IV) techniques are used in 20% of all cases. Since neglecting endogeneity problems may induce bias, IV estimations are chosen as the reference category.[20]

Regarding data characteristics, country data are treated as the reference because they incorporate all substitution possibilities for which lower level data cannot account. To be consistent with country data, the reference category for the data structure is time series estimation. Regarding data on production factors, labour differs in productivity due to differences in workers' human capital levels. Estimates relying on some type of quality adjustment applied to the aggregate labour input are chosen as the reference category, as they account for such considerations.

Econometric model

Using the variables described above, our aim is to explain the heterogeneity in existing estimates of σ by drawing on a regression framework. The general econometric model can be written as:

$$\tilde{\sigma}_{ij} = \sigma_0 + \sum_{k=1}^{K} \beta_k x_{kij} + \varepsilon_{ij} \tag{3}$$

with $\tilde{\sigma}_{ij}$ being the estimate $i = 1, 2, \ldots, n_j$ of σ in study $j = 1, 2, \ldots, J$. σ_0 is the constant term of the regression model with regressors x_k, $k = 1, \ldots, K$ that are supposed to explain the heterogeneity in the estimates of σ. The regression coefficients are denoted by β_k and ε_{ij} represents the error term. As our variables are categorical, we create dummies for each of these categories. If, for instance, in the 'univariate' case a variable has L categories, $L-1$ dummies D^l with $l = 1, 2, \ldots, L-1$ are included in the regression model:

$$\tilde{\sigma}_{ij} = \sigma_0 + \beta_1 D_{ij}^1 + \beta_2 D_{ij}^2 + \cdots + \beta_{L-1} D_{ij}^{L-1} + \varepsilon_{ij} \tag{4}$$

Category L is the so-called reference category. As outlined above, we choose all reference categories such that they represent the most reliable estimation of σ from a theoretical and methodological perspective. The model consisting only of these reference categories is the reference model. The advantage of this type of procedure is that it allows us to interpret the constant term of the regression model as the elasticity of substitution one would *expect*

[18] Note that choosing another category (e.g. 'short-run' estimates) as the baseline category would not alter the implications of our results.

[19] Our results indicate that different estimation methods (least squares or other techniques) do not affect econometric results of primary studies in important ways. Thus, our results imply that the chosen estimation methods have only modest effects on the estimated elasticity.

[20] The most challenging task for IV regression is to find an exogenous and relevant instrument. If the instrumental variable itself is endogenous, the IV estimator is not consistent. Although we are aware of this potential drawback, we choose IV estimations as the reference category as we believe this approach to be superior to ignoring a likely simultaneous equation bias.

to estimate using a model with the specification corresponding to the reference categories.[21] Thus, an appealing feature of a meta-regression is that it allows us to obtain such a meta-estimate although no study in our sample fulfils all criteria of the reference specification.

Another important issue in meta-regression is precision weighting. In general, more precise estimates should have higher weight in meta-regression analysis since they are, on average, closer to the population value of the estimated parameter.[22] Therefore, our preferred estimation approach relies on inverse standard error weighting.

Results

Table 2 shows the meta-regression results for weighted least squares (WLS) and random effects (RE) models. Models (1) and (2) pool all observations, that is, industry specific as well as aggregate economy estimates of σ. Models (3) and (4) introduce industry dummies. This approach allows us to test explicitly for the difference between the elasticity of substitution of each industry and the aggregate economy. The inclusion of industry dummies has almost no effect on the estimates of other coefficients. Therefore, we will not discuss the models with and without industry dummies separately.

Our results for the dummies capturing the choice of different estimation equations are mostly consistent with the evidence provided by León-Ledesma *et al.* (2010). The regression coefficients for the FOC of capital, the combination of the FOCs of labour and capital (FOC combined) and factor shares are always negative and significant ($P < 0.01$), indicating that estimates of σ relying on such functions tend to be lower than estimates obtained from system estimations. According to the random effects models, the largest effect occurs from direct estimations of the production function. The coefficient of the reverse FOC of labour suggests that such estimations lead to higher estimates of σ compared to system estimations. Again, the difference is significant for the RE estimation only. For all other FOC variants, the difference compared to using an equation system seems to be small and insignificant. The point estimates for the reverse variants of the FOCs are larger than the point estimates for their non-reversed counterparts. The point estimates are also consistent with the observation already made by Berndt (1976). Namely, estimates of σ tend to be higher when using the FOC of labour instead of the FOC of capital. Linear approximations of the production function seem to lead to more diverse results – their point estimates are large but imprecise. The sign of the coefficient switches between WLS

[21] This can be formally illustrated in the following fashion:

$$\hat{\hat{\sigma}}_{ij} = \hat{\sigma}_0 + \hat{\beta}_1 D^1_{ij} + \hat{\beta}_2 D^2_{ij} + \cdots + \hat{\beta}_{L-1} D^{L-1}_{ij}$$

The regression coefficient $\hat{\beta}_l$ represents the estimated marginal effect of category $l \neq L$ relative to the reference category. In other words, $\hat{\beta}_l$ indicates the expected change in the estimated elasticity if the estimation specification of a study deviates in that regard from the reference, c. p. In the reference model, all dummies are zero:

$$x_{ij} = L \Longrightarrow D^1_{ij} = D^2_{ij} = \cdots = D^{L-1}_{ij} = 0 \Longrightarrow \hat{\hat{\sigma}}_{ij} = \hat{\sigma}_0$$

The estimate of the constant term $\hat{\sigma}_0$ yields the estimated value of σ for the reference category. This result generalizes to multiple regression. In this case, $\hat{\sigma}_0$ represents the estimated value of σ for a study making use of the reference choices for all variables simultaneously.

[22] Of course, this is only true if the estimators are unbiased.

TABLE 2

Results of WLS and RE estimations using inverse standard error weighting

	(1) WLS	(2) RE	(3) WLS	(4) RE
System (Ref.)	–	–	–	–
Production function	−0.0657	−0.445**	−0.0694	−0.399**
	(0.273)	(0.216)	(0.270)	(0.202)
FOC capital	−0.194***	−0.132***	−0.178***	−0.123***
	(0.0575)	(0.0340)	(0.0527)	(0.0332)
FOC combined	−0.312***	−0.184***	−0.289***	−0.176***
	(0.0734)	(0.0395)	(0.0670)	(0.0383)
FOC labour	0.0109	0.00212	0.0154	0.0108
	(0.0697)	(0.0391)	(0.0672)	(0.0381)
Linear approximation	0.336	−0.214	0.339	−0.169
	(0.231)	(0.175)	(0.235)	(0.164)
Rev. FOC capital	−0.00232	0.00264	0.0162	0.0119
	(0.0769)	(0.0749)	(0.0793)	(0.0734)
Rev. FOC combined	−0.0906	0.00434	−0.0713	0.0118
	(0.0839)	(0.0713)	(0.0840)	(0.0692)
Rev. FOC labour	0.178	0.180***	0.187	0.188***
	(0.151)	(0.0647)	(0.146)	(0.0633)
Factor shares	−0.150***	−0.141***	−0.147***	−0.140***
	(0.0477)	(0.0297)	(0.0465)	(0.0290)
Factor-augmenting, Box–Cox (Ref.)	–	–	–	–
Factor-augmenting, constant growth	0.314***	0.285***	0.304***	0.280***
	(0.0301)	(0.0412)	(0.0287)	(0.0399)
Factor-augmenting, other	0.314***	0.151	0.304**	0.153
	(0.118)	(0.155)	(0.118)	(0.137)
Hicks-neutral, constant growth	0.438***	0.436***	0.431***	0.429***
	(0.0841)	(0.0763)	(0.0850)	(0.0741)
No dynamics	0.421***	0.565***	0.407***	0.542***
	(0.0606)	(0.0607)	(0.0591)	(0.0585)
Levels (Ref.)	–	–	–	–
Growth rates	−0.153***	−0.123***	−0.152***	−0.121***
	(0.0346)	(0.0212)	(0.0323)	(0.0208)
IV (Ref.)	–	–	–	–
Non-IV	−0.118***	−0.0648***	−0.114***	−0.0668***
	(0.0442)	(0.0226)	(0.0415)	(0.0221)
Least squares (Ref.)	–	–	–	–
Other method	0.0339	0.0101	0.0258	0.00763
	(0.0330)	(0.0217)	(0.0321)	(0.0211)
Quality-adjusted labour (Ref.)	–	–	–	–
Unadjusted labour	0.0586**	0.0261	0.0733**	0.0417
	(0.0282)	(0.0253)	(0.0297)	(0.0262)
Country (Ref.)	–	–	–	–
Firm level	−0.449***	−0.344**	−0.496***	−0.393***
	(0.0772)	(0.153)	(0.0721)	(0.139)
Industry level	−0.318***	−0.107*	−0.367***	−0.178***
	(0.0479)	(0.0598)	(0.0567)	(0.0652)

(*continued*)

TABLE 2

(Continued)

	(1) WLS	(2) RE	(3) WLS	(4) RE
Time series (Ref.)	–	–	–	–
Cross section	0.254***	0.194**	0.267***	0.198***
	(0.0591)	(0.0786)	(0.0580)	(0.0749)
Panel	0.212*	0.133***	0.236*	0.178***
	(0.127)	(0.0464)	(0.128)	(0.0540)
Mark-up (Ref.)	–	–	–	–
No mark-up	−0.0439	−0.169***	−0.0464	−0.175***
	(0.0480)	(0.0536)	(0.0472)	(0.0503)
Journal article (Ref.)	–	–	–	–
Monograph	−0.0601	−0.00610	−0.0454	−0.00159
	(0.0573)	(0.118)	(0.0535)	(0.103)
Working paper	−0.0579*	−0.0505	−0.0459	−0.0406
	(0.0304)	(0.0732)	(0.0312)	(0.0646)
Long-run (Ref.)	–	–	–	–
Short-run	−0.189*	−0.156***	−0.194**	−0.165***
	(0.0998)	(0.0285)	(0.0919)	(0.0281)
Theoret. long-run/emp. short-run	0.0479	−0.0745***	0.0325	−0.0816***
	(0.0722)	(0.0239)	(0.0660)	(0.0237)
σ_0	0.642***	0.707***	0.648***	0.717***
	(0.105)	(0.0837)	(0.0995)	(0.0770)
Observations	2,419	2,419	2,419	2,419
R^2	0.612		0.630	
Industry dummies	No	No	Yes	Yes
σ_0 lower bound	0.434	0.543	0.450	0.566
σ_0 upper bound	0.851	0.871	0.846	0.868
Number of studies		77		77

Notes: WLS with clustered standard errors in parentheses; ***$P < 0.01$, **$P < 0.05$, *$P < 0.1$.

and RE estimates. Overall, however, the results are consistent with our expectations given the Monte Carlo evidence provided by León-Ledesma *et al.* (2010).

More important than the choice of the estimating equation seems to be the modelling of technological change. All results consistently show that a less flexible modelling of technological change results in higher estimates of the elasticity of substitution. According to models (1)–(4), choosing a constant growth rate instead of the Box–Cox transformation for factor-augmenting technological change increases estimates of σ by approximately 0.29–0.31. Allowing only for Hicks-neutral change increases the estimates of σ by approximately 0.43–0.44, and ignoring technological change altogether increases the estimate of σ by 0.41–0.57. These large effect sizes are accompanied by small standard errors, underscoring the strength of our evidence. Therefore, differences in specifications of the technological dynamics account for the major differences in the estimation results reported in the literature and can be considered a crucial design choice of empirical papers. On average, specifications belonging to the category 'factor-augmenting, other' result in higher estimates, although not significantly for the RE models.

Estimations based on growth rates and the use of techniques to address endogeneity are observed to be associated with lower estimates of σ. We do not find relevant effects of utilizing least squares instead of other estimation methods. The point estimate of unadjusted labour is positive but small. However, it is noteworthy that adjusting the aggregate labour input is not equivalent to estimating skill-specific elasticities of substitution. The fact that we do not observe a relevant impact might, therefore, reflect shortcomings of common approaches to adjusting labour quality using aggregate data.

Turning to the data structure, we observe that the use of firm- and industry-level data yields systematically different estimates compared to those resulting from the use of country-level data. In particular, firm-level data lead to much lower estimates of σ. This could reflect that firm- and industry-level estimates might not capture substitution possibilities at higher levels of aggregation. Estimates obtained from panel and cross-sectional data are higher than those obtained from time series data. According to the estimated effect sizes, there is evidence that these data structures considerably alter estimates of σ. Allowing for a mark-up is observed to decrease estimates of σ in the RE models. Hence, the assumption of whether the zero-profit condition holds when deriving estimation equations may be important for estimation outcomes. Our results do not give any reason to believe that estimates reported in working papers or monographs differ systematically from those reported in journal articles.

Estimates of the 'short-run' parameters reported in Table 2 show that the substitution between capital and labour is more elastic in the long run than in the short run. Depending on the model, the short-run elasticity is estimated to be approximately 0.16–0.19 lower than the long-run elasticity. Moreover, estimates of the long-run elasticity based on unadjusted (short-run) data are between short-run and long-run estimates. Thus, the application of suitable econometric techniques or data transformations when estimating long-run elasticities, as recently done by Chirinko and Mallick (2017), appears to be an important modelling decision.

Finally, σ_0 reports our meta-estimates of σ for the discussed reference categories. For the models (3) and (4) that consider industry differences, the WLS estimate of 0.65 is slightly smaller than the RE estimate of 0.72. The table also reports lower and upper bounds of σ_0 (95% CI). None of the upper bounds exceed 0.871. Based on this evidence, a Cobb–Douglas production function can be rejected at the 5% significance level. In addition, Figure 2 shows the deviation of each industry from the aggregate economy. As can be seen, only a few significant differences can be identified. In particular, the tobacco, apparel, machinery, transportation equipment and miscellaneous manufacturing industries differ significantly from the aggregate economy.[23] Our results indicate that all of these industries have a higher elasticity of factor substitution than the economy in the aggregate. For the other 15 industries, the confidence interval includes zero. Hence, a deviation of zero cannot be ruled out.

Robustness and Extensions

In addition to the specification of weighted least squares and random effects models in the previous section, we show results for fixed effects (FE) estimates as a robustness check

[23] Table 2 in the online appendix provides the complete list of included industries according to the Standard Industrial Classification (SIC).

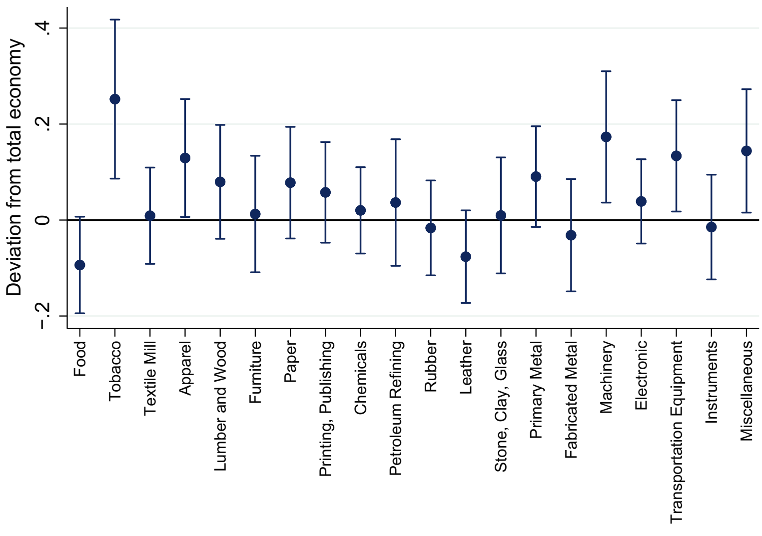

Figure 2 Deviations of industry-specific substitution elasticities compared to the aggregate economy substitution elasticity based on WLS estimates with 95% CI

in Table 3. The results without and with industry dummies are shown in models (5) and (6) respectively. Applying FE in meta-studies has two drawbacks, which is why they serve as robustness checks only. First, it is not possible to estimate effects for study-invariant factors, such as the publication type (journal, working paper or monograph). Second, it is not possible to estimate a meta-elasticity. Qualitatively, our primary results remain stable. The FE models also indicate that approaches other than equation system estimation may lead to lower estimates of σ or have no effect, except for the reverse FOC for labour. Contrary to our primary results, both models suggest that the use of linear approximations of the production function to estimate σ leads to significantly lower estimates, which is consistent with the simulation results obtained by León-Ledesma *et al.* (2010). The already large coefficients of production function estimations in the RE models of Table 2 become even larger. The crucial role of technological dynamics is confirmed by the FE models. All coefficients are close to the results of the RE models, except for the coefficient of 'factor augmenting, other', which drops out of the regressions due to a lack of variation within studies. The coefficients for growth rates, IV, least squares and the quality adjustment of labour remain virtually unchanged. According to the FE models, estimates based on industry- and firm-level data do not significantly differ from those based on country-level data, although the size of the coefficients is similar to the RE results. However, the loss in precision could be driven by the low variability of data sets within studies. Cross-sectional and panel results again lead to higher estimates of σ. In the FE models, the coefficient of the mark-up is not significant, which could again be driven by the low variability of mark-up specifications within studies. Finally, as before, our FE results indicate that short-run estimates lead to lower estimates of σ. The magnitude of the coefficients is almost the same as for the RE regressions.

In summary, our results show that the choice of the estimation equation and the specification of technological dynamics crucially affect estimates of σ. In particular, we showed that

TABLE 3

Results of FE estimations using inverse standard error weighting

	(5) *FE*	*(6)* *FE*
System (Ref.)	–	–
Production function	−0.732***	−0.726***
	(0.274)	(0.267)
FOC capital	−0.117***	−0.104***
	(0.0357)	(0.0349)
FOC combined	−0.166***	−0.151***
	(0.0423)	(0.0413)
FOC labour	0.0178	0.0303
	(0.0414)	(0.0405)
Linear approximation	−0.452**	−0.435**
	(0.222)	(0.217)
Rev. FOC capital	0.0226	0.0362
	(0.0761)	(0.0742)
Rev. FOC combined	0.0360	0.0495
	(0.0752)	(0.0734)
Rev. FOC labour	0.199***	0.212***
	(0.0661)	(0.0645)
Factor shares	−0.133***	−0.130***
	(0.0304)	(0.0296)
Factor-augmenting, Box–Cox (Ref.)	–	–
Factor-augmenting, constant growth	0.317***	0.315***
	(0.0456)	(0.0444)
Factor-augmenting, other		
Hicks-neutral, constant growth	0.442***	0.435***
	(0.0808)	(0.0788)
No dynamics	0.585***	0.559***
	(0.0696)	(0.0684)
Levels (Ref.)	–	–
Growth rates	−0.119***	−0.116***
	(0.0213)	(0.0208)
IV (Ref.)	–	–
Non-IV	−0.0647***	−0.0667***
	(0.0236)	(0.0230)
Least squares (Ref.)	–	–
Other method	0.00723	0.00473
	(0.0226)	(0.0221)
Quality-adjusted labour (Ref.)	–	–
Unadjusted labour	0.0387	0.0640**
	(0.0270)	(0.0284)
Country (Ref.)	–	–
Firm level	−0.217	−0.307
	(0.378)	(0.370)
Industry level	−0.0290	−0.114
	(0.0839)	(0.0875)
Time series (Ref.)	–	–

(*continued*)

TABLE 3

(Continued)

	(5) *FE*	*(6)* *FE*
Cross section	0.283***	0.301***
	(0.0928)	(0.0914)
Panel	0.0855*	0.136**
	(0.0495)	(0.0568)
Mark-up (Ref.)	–	–
No mark-up	−0.102	−0.104
	(0.0691)	(0.0674)
Journal article (Ref.)	–	–
Monograph	*collinear*	*collinear*
Working paper	*collinear*	*collinear*
Long-run (Ref.)	–	–
Short-run	−0.154***	−0.163***
	(0.0285)	(0.0280)
Theoret. long-run/emp. short-run	−0.0905***	−0.102***
	(0.0241)	(0.0237)
Observations	2,411	2,411
R^2	0.126	0.170
Industry dummies	No	Yes
Number of studies	69	69

Notes: $^{***}P<0.01$, $^{**}P<0.05$, $^{*}P<0.1$.

deviating from a factor-augmenting Box–Cox specification by assuming constant growth rates of technology or no dynamics at all substantially alters the estimation outcomes. Allowing for a mark-up when estimating σ based on first-order conditions might be a favourable strategy.

Regarding the reference estimates of the long-run, aggregate σ, two main conclusions can be drawn. First, based on our meta-estimates, the long-run substitution elasticity between capital and labour is in the range of 0.45–0.87. Second, a Cobb–Douglas production function implying a unitary elasticity can be rejected based on our data.[24] Furthermore, most industrial estimates do not deviate significantly from the estimate of the aggregate elasticity.

V. Conclusion

In view of approximately constant factor shares and supportive empirical evidence from certain influential econometric studies, production functions of the Cobb–Douglas type have been a common choice for describing the aggregate output of the US economy in the past. This is in contrast to the estimated values of the elasticity of substitution in the empirical literature obtained using a CES framework. The majority of studies suggests that $\sigma<1$. However, estimation results are characterized by substantial heterogeneity within

[24] Of course, one might disagree with our reference categories. However, the choice of the reference category does not alter the regression results. It is, therefore, possible to calculate the point estimate of σ_0 for any combination of reference categories. For instance, if one prefers model (3) but believes that estimating in growth rates is superior to estimating in levels, then $0.648-0.152=0.496$ would be the meta-estimate of the aggregate σ.

and across studies. In past research, several conjectures have been made to explain these differences. By applying meta-regression techniques, we present the first rigorous quantitative assessment to jointly test multiple potential factors influencing estimates of σ. Furthermore, defining suitable reference categories for our regressors allows us to obtain meta-estimates of σ based on our meta-sample.

Our results indicate that differences in the specification of technological dynamics substantially affect the estimation outcomes. For instance, neglecting technological dynamics increases estimates of σ by 0.41–0.57, on average, compared to those resulting from modelling factor-augmenting technological change with the most flexible Box–Cox transformation. We also find evidence for a crucial role of the choice of the estimation equation. Thus, the use of single-equation approaches results, on average, in lower estimated elasticities compared to equation system estimations.

Turning to our meta-estimate of σ, all estimation results clearly indicate values below unity. The interval estimates of our preferred models (3) and (4) indicate plausible values of the long-run σ of the aggregate economy in the range of 0.45–0.87. This finding suggests that a Cobb–Douglas production function is unlikely to be a good representation of the US economy. Only certain industry estimates deviate significantly from the aggregate estimate of σ.

Appendix

Histogram and funnel plot for single industries

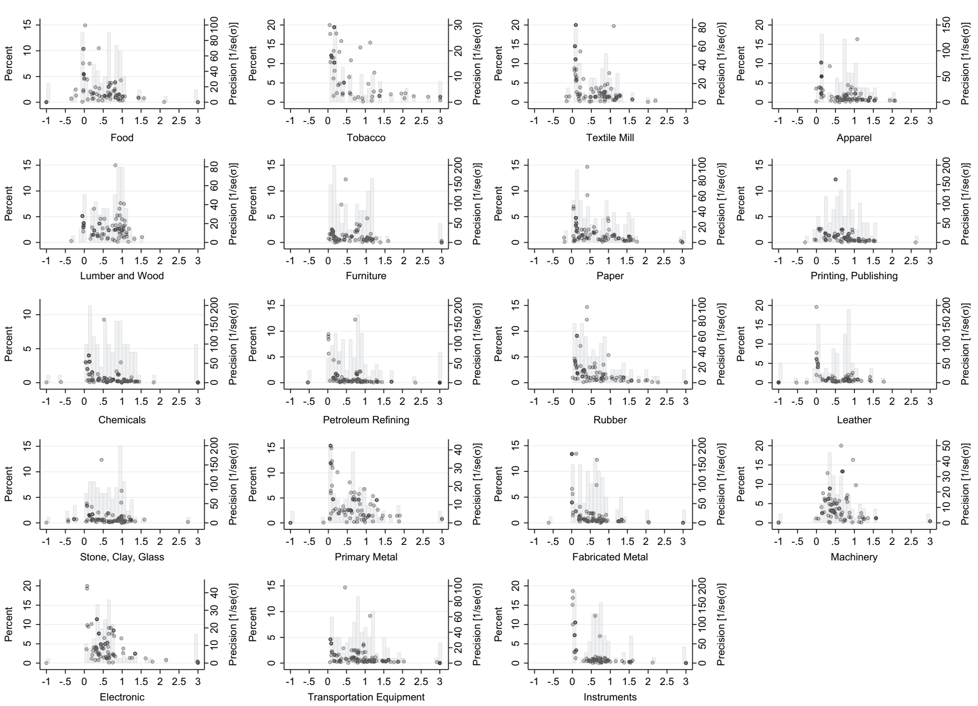

Figure A1 Open-ended histogram and funnel plot, depicting collected estimates for the elasticity of factor substitution for each industry

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Supporting Information

Additional supporting information may be found in the online version of this article:

Appendix A. Derivation and assembly of FOC variants.
Appendix B. Calculation of standard errors.
Appendix C. SIC Classification of Industries.
Appendix D. Study sample.